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ON ARMENDARIZ-LIKE PROPERTIES

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Abstract

In this paper, we attempt to construct a class of Armendariz-Like properties. We investigate the transfer of the Armendariz-Like properties to trivial ring extensions to localization and direct product of rings, and then generate new families of rings with zero-divisors subject to some given Armendariz-like properties. The article includes a brief discussion of the scope and precision of our results.

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1 Introduction

Throughout this paper, all rings are associative with identity elements, and all modules are unital. It is suitable to use "local" to refer to (not necessarily Noetherian) ring with a unique maximal ideal. A subring of a ring need not have the same unit.

Let R be a ring. The content $C(f)$ of a polynomial $f \in R[x]$ is the ideal of R generated by all coefficients of f. One of its properties is that $C(.)$ is semi-multiplicative, that is $C(fg) \subseteq C(f)C(g)$, and a polynomial $f \in R[x]$ is said to be Gaussian over R if $C(fg) = C(f)C(g)$, for every polynomial $g \in R[x]$. A polynomial $f \in R[x]$ is Gaussian provided $C(f)$ is locally principal by [9, Remark 1.1]. A ring R is said to be a Gaussian ring if $C(fg) = C(f)C(g)$ for any polynomials f, g with coefficients in R. A domain is Gaussian if and only if it is a Prüfer domain. See for instance [2, 5, 6, 8, 9].

Definition 1.1. (Armendariz-like properties).

1) A ring R is called a reduced ring if it has no non-zero nilpotent elements.

2) A ring R is called an Armendariz ring if whenever the product of two polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{i=0}^{m} b_i x^i \in R[x]$ satisfies $fg = 0$, we have $C(f)C(g) = 0$ (that is $a_i b_j = 0$ for every i and j).

3) A ring R is called a nilArmendariz ring if whenever the product of two polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{i=0}^{m} b_i x^i \in R[x]$ satisfies $fg \in nil(R)[x]$, we have $C(f)C(g) \in nil(R)$ (that is $a_i b_j \in nil(R)$ for every i and j).

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4) A ring R is called a weakArmendariz ring if whenever the product of two polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{i=0}^{m} b_i x^i \in R[x]$ satisfies $fg=0$, we have $C(f)C(g) \in nil(R)$, (that is $a_i b_j \in nil(R)$ for every i and j).

In [16], Rege and Chhawchharia (1997), introduced the notion of an Armendariz ring. E. Armendariz proved in [3] that a reduced ring satisfies this condition. Armendariz rings are thus a generalization of reduced rings. It is easy to see that subring of Armendariz rings are also Armendariz. E. Armendariz [3, Lemma 1] noted that any reduced ring is an Armendariz ring. Also, D. D. Anderson and V. Camillo [2], show that a ring R is Gaussian if and only if every homomorphic image of R is Armendariz. See for instance [2, 3, 14, 16].

In [1], Ramon Antoine (2008), introduced nilArmendariz rings. Armendariz ring is nilArmendariz [1, Proposition 2.7]. It is easy to see that subring of nilArmendariz ring is also nilArmendariz. In [15], Lui and Zhao (2006), introduced weakArmendariz rings as a generalization of Armendariz. It is clear that subring of weakArmendariz ring is also weakArmendariz. Obviously, nilArmendariz rings are weakArmendariz rings. The following diagram of implications summarizes the relations between them (See for instance [1, 15]):

$$\text{Reduced} \Rightarrow \text{Armendariz} \Rightarrow \text{nilArmendariz} \Rightarrow \text{weakArmendariz.}$$

In [1], it is proved that each one of above conditions implies the following next one . Also there are examples given to show that in general, the implications cannot be reversed. There is non-reduced Armendariz ring by [16, Proposition 2.1]. There is a non-Armendariz nilArmendariz ring by [1, Examples 4.9]. But, we do not know so far any example of a weakArmendariz ring that is not nilArmendariz. Recall that a ring R is semicommutative if for each $a,b \in R$, $ab=0$ implies $aRb=0$. Semicommutative ring is nilArmendariz [15, Proposition 3.3]. Thus weakArmendariz rings and nilArmendariz rings are a common generalization of semicommutative rings and Armendariz rings.

Let A be a ring, E be an A-module and $R := A \propto E$ be the set of pairs (a,e) with pairwise addition and multiplication given by $(a,e)(b,f)=(ab, af+be)$. R is called the trivial ring extension of A by E (also called the idealization of E over A). Considerable work has been concerned with trivial ring extension. Part of it has been summarized in Glaz's book [7], and Huckaba's book (where R is called the idealization of E by A) [**?**]. See for instance [7, 11, 12].

The aim of this paper is to present a class of Armendariz-like properties. We investigate the transfer of the Armendariz-Like properties to trivial ring extensions, to localization and to direct product of rings. Our results generate new and original examples which enrich the current literature with new families of Armendariz-Like properties with zero-divisors.

2 Main Results

This section develops a result of the transfer of the Armendariz-Like properties of trivial ring extensions. And so we will construct a new class of Armendariz-Like properties.

Theorem 2.1. *Let A be a ring, E be a nonzero A-module, and let $R := A \propto E$. Then:*

1) R is a weakArmendariz ring if and only if so is A.

2) R is a nilArmendariz ring if and only if so is A.

3) Assume that (A,M) is a local ring and E an A-module such that $ME = 0$. Then, R is an Armendariz ring if and only if so is A.

4) If R is a semicommutative ring, then so is A.

5) R is never a reduced ring.

Proof. 1) If R is a weakArmendariz ring, then so is A since A is a subring of R. Conversely, assume that A is a weakArmendariz ring. Let $f = \sum_{i=0}^{n}(a_i, e_i)x^i$ and $g = \sum_{i=0}^{m}(b_i, f_i)x^i$ be two polynomials in $R[x]$ such that $fg = 0$, where n and m are positive integers. Set $f_A := \sum_{i=0}^{n} a_i x^i$ and $g_A := \sum_{i=0}^{m} b_i x^i$ be two polynomials of $A[x]$. We have $f_A g_A = 0$ since $fg = 0$. Hence $C_A(f_A)C_A(g_A) \in nil(A)$ since A is a weakArmendariz ring. But, $C(f)C(g) = (C_A(f_A)C_A(g_A), c)$, where $c \in E$. Therefore, $C(f)C(g) \in nil(R)$ (since $(a,c) \in nil(R)$ if and only if $a \in nil(A)$).

2) If R is a nilArmendariz ring, then so is A since A is a subring of R. Conversely, assume that A is a nilArmendariz ring. Let $f = \sum_{i=0}^{n}(a_i, e_i)x^i$ and $g = \sum_{i=0}^{m}(b_i, f_i)x^i$ be two polynomials in $R[x]$ such that $fg \in nil(R)[x]$, where n and m are positive integers. Set $f_A := \sum_{i=0}^{n} a_i x^i$ and $g_A := \sum_{i=0}^{m} b_i x^i$ be two polynomials of $A[x]$. We have $f_A g_A \in nil(A)[x]$ since $fg \in nil(R)[x]$. Hence $C_A(f_A)C_A(g_A) \in nil(A)$ since A is a nilArmendariz ring. But, $C(f)C(g) = (C_A(f_A)C_A(g_A), c)$, where $c \in E$. Therefore, $C(f)C(g) \in nil(R)$ (since $(a,c) \in nil(R)$ if and only if $a \in nil(A)$).

3) By [4, Theorem 2.1].

4) If R is a semicommutative ring, then so is A since A is a subring of R.

5) R is never a reduced ring since $0 \neq (0 \propto E) \subseteq nil(R)$. □

The following example shows that weak(nil)Armendariz rings may not be semicommutative rings and shows that in general, the implication of Theorem 2.1(4) cannot be reversed.

Recall that a ring R is called reversible if $ab = 0$ implies $ba = 0$ for $a, b \in R$. Reversible rings are semicommutatives by [15, Lemma 1.4].

Example 2.2. Let $\mathbb{H}$ be the Hamilton quaternions over the real number field, $A := \mathbb{H} \propto \mathbb{H}$ and set $R := A \propto A$. Then:

1) A is semicommutative (since it is reversible by [13, Example 1.7]).

2) A is not reduced by Theorem 2.1(5).

3) R is not semicommutative by [13, Example 1.7].

R is nilArmendariz by Theorem 2.1(2).

The following example shows that weak(nil)Armendariz rings may not be Armendariz rings, and shows that Armendariz rings may not be reduced rings.

Also, it shows that the trivial ring extension of an Armendariz ring by itself is not always an Armendariz ring.

Example 2.3. Let K be a field, $A := K \propto K$, and let $R := A \propto A$. Then :

1) A is an Armendariz ring.

2) A is not a reduced ring.

3) R is not an Armendariz ring.

4) R is a nilArmendariz ring.

Proof. 1) A is a Gaussian ring by [9, Remark 1.1]. In particular, A is an Armendariz ring.

2) Hold by direct application of Theorem 2.1.

3) Our aim is to show that R is not an Armendariz ring. Let $f = ((0,1),(0,0)) + ((0,0),(1,0))x$ and $g = ((0,1),(0,0)) + ((0,0),(-1,0))x$ be a two polynomials in $R[x]$. We easily check that $fg = 0$ and $C(f)C(g) = [R((0,1),(0,0)) + R((0,0),(1,0))][R((0,1),(0,0)) + R((0,0),(-1,0))] \neq 0$.

4) Hold by direct application of Theorem 2.1. □

Now, we study the relationship between an idempotent elements and Armendariz-like properties.

Theorem 2.4. *Let R be a ring and e an idempotent central element of R. Then:*

1) R is a reduced ring if and only if so are eR and $(1-e)R$.

2) R is an Armendariz ring if and only if so are eR and $(1-e)R$.

3) R is a nilArmendariz ring if and only if so are eR and $(1-e)R$.

4) R is a weakArmendariz ring if and only if so are eR and $(1-e)R$.

Proof. 1) It is easy to see that eR and $(1-e)R$ are subrings of R, hence if R is a reduced ring, then so are eR and $(1-e)R$. Conversely, assume that eR and $(1-e)R$ are reduced rings and let $x \in R$ such that $x^n = 0$. Hence, $(ex)^n = 0$ and $((1-e)x)^n = 0$, thus $(ex) = 0$ and $((1-e)x) = 0$ since eR and $(1-e)R$ are reduced rings and so $x = 0$. This means that R is a reduced ring.

2) , 3) and 4). In [10], it is proved 2). The same is proved for 4) in [15], and also true for a nilArmendariz rings. □

Now, we study the localization of Armendariz-Like properties.

Theorem 2.5. *Let R be a ring. Then:*

1) a) Assume that R is a weakArmendariz ring and S is a multiplicative subset of R. Then $S^{-1}R$ is a weakArmendariz ring.

b) A ring R is a weakArmendariz if and only if so is R_M for each maximal ideal M of R.

2) a) Assume that R is a nilArmendariz ring and S is a multiplicative subset of R. Then $S^{-1}R$ is a nilArmendariz ring.

b) A ring R is a nilArmendariz if and only if so is R_M for each maximal ideal M of R.

3) a) Assume that R is an Armendariz ring and S is a multiplicative subset of R. Then $S^{-1}R$ is an Armendariz ring.

b) A ring R is an Armendariz ring if and only if so is R_M for each maximal ideal M of R.

4) Assume that R is a ring and S is a multiplicative subset of R which is contained in $R \backslash Z(R)$. Then R is a reduced ring if and only if so is $S^{-1}R$.

5) Assume that R is a ring and S is a multiplicative subset of R which is contained in $R \backslash Z(R)$. Then R is a semicommutative ring if and only if so is $S^{-1}R$.

Proof. 1) a) Without loss of generality, we may consider the polynomials of the form $S^{-1}f$ and $S^{-1}g$, where $f = \sum_{i=0}^{n} a_i x^i$ and $g = \sum_{i=0}^{m} b_i x^i \in R[x]$, such that $S^{-1}fS^{-1}g = 0$. Hence there exists $t \in S$ such that $tfg = 0$ and so $tC_R(f)C_R(g) = C_R(tf)C_R(g) \in nil(R)$ since R is a weakArmendariz ring. Then we have :

$$\begin{aligned} C_{S^{-1}R}(S^{-1}f)C_{S^{-1}R}(S^{-1}g) &= S^{-1}(C_R(f))S^{-1}(C_R(g)) \\ &= S^{-1}(C_R(f)C_R(g)) \\ &= S^{-1}(tC_R(f)C_R(g)) \in nil(S^{-1}R) \end{aligned}$$

Therefore, $S^{-1}R$ is a weakArmendariz ring.

b) If R is a weakArmendariz ring, then so is R_M for each maximal ideal M of R by (a). Conversely, assume that R_M is a weakArmendariz ring for each maximal ideal M and let $f, g \in R[x]$ such that $fg = 0$. Then $C(fg)_M = 0$ and so $[C(f)C(g)]_M = (C(f)_M C(g)_M) \in nil(R_M)$ for each maximal ideal M since R_M is a weakArmendariz ring. Therefore, $C(f)C(g) \in nil(R)$.

2) a) Without loss of generality, we may consider the polynomials of the from $S^{-1}f$ and $S^{-1}g$ where $f = \sum_{i=0}^{n} a_i x^i$ and $g = \sum_{i=0}^{m} b_i x^i \in R[x]$, such that $S^{-1}fS^{-1}g \in nil(S^{-1}R)[x]$. Hence there exists $t \in S$ such that $tfg \in nil(R)[x]$ and so, $tC_R(f)C_R(g) = C_R(tf)C_R(g) \in nil(R)$ since R is a nilArmendariz ring. Then we have :

$$\begin{aligned} C_{S^{-1}R}(S^{-1}f)C_{S^{-1}R}(S^{-1}g) &= S^{-1}(C_R(f))S^{-1}(C_R(g)) \\ &= S^{-1}(C_R(f)C_R(g)) \\ &= S^{-1}(tC_R(f)C_R(g)) \in nil(S^{-1}R) \end{aligned}$$

Therefore, $S^{-1}R$ is a nilArmendariz ring.

b) If R is nilArmendariz ring, then so is R_M for each maximal ideal M of R by a). Conversely, assume that R_M is a nilArmendariz ring. For each maximal M and let $f, g \in R[x]$ such that $fg \in nil(R)[x]$. Then $C(fg)_M \in nil(R_M)$ and so $[C(f)C(g)]_M = (C(f)_M C(g)_M) \in nil(R_M)$ for each maximal ideal M since R_M is a nilArmendariz ring. Therefore, $C(f)C(g) \in nil(R)$.

3) a) By [4, Theorem 2.8.(1)].

b) By [4, Theorem 2.8.(2)].

4) Assume that R is a reduced ring and let $(a/t) \in S^{-1}R$ such that $(a/t)^n = 0$. Hence, $(a^n/t^n) = 0$ and so there exists $\acute{t} \in S$ such that $a^n\acute{t} = 0$; thus $a^n = 0$, since $S \subseteq R \backslash Z(R)$ and so $a = 0$ since R is a reduced ring. Then $(a/t) = 0$ which means that $S^{-1}R$ is reduced.

Conversely, assume that $S^{-1}R$ is a reduced ring and let $x \in R$ such that $x^n = 0$. Hence, $x^n = (x/1)^n = 0$ and then $x/1 = 0$ since $S^{-1}R$ is a reduced ring, which means that $x = 0$. Hence, R is a reduced ring.

5) Assume that R is a semicommutative ring and let a/t and $\acute{a}/\acute{t} \in S^{-1}R$ such that (a/t) $(\acute{a}/\acute{t}) = 0$. Thus $a\acute{a}/t\acute{t} = 0$ and so there exists $\acute{\acute{t}} \in S$ such that $a\acute{a}\acute{\acute{t}} = 0$. Hence, $a\acute{a} = 0$ (since $\acute{\acute{t}} \in S \subseteq R \backslash Z(R)$) and so $aR\acute{a} = 0$ (since R is a semicommutative ring). Therefore, $(a/t)S^{-1}R(\acute{a}/\acute{t}) = 0$ and so $S^{-1}R$ is a semicommutative ring.

Conversely, assume that $S^{-1}R$ is a semicommutative ring and let $x, y \in R$ such that $xy = 0$. Hence, $(xy/1) = (x/1)(y/1) = 0$ and so $(x/1)S^{-1}R(y/1) = 0$ since $S^{-1}R$ is a semicommutative ring. Therefore, $xRy = 0$ and then R is a semicommutative ring. □

By Theorem 2.5 and since each domain is Armendariz, we have:

Corollary 2.6. *A local domain is an Armendariz ring. In particular, it is a weak(nil)Armendariz ring.*

Corollary 2.7. *Let R be a ring. Then:*
1) $R[x]$ is a weakArmendariz ring if and only if so is $R[x,x^{-1}]$.
2) $R[x]$ is a nilArmendariz ring if and only if so is $R[x,x^{-1}]$.
3) $R[x]$ is an Armendariz ring if and only if so is $R[x,x^{-1}]$.

Proof. Let $S := \{1,x,x^2,...\}$ be a multiplicatively closed subset of $R[x]$. Since $R[x,x^{-1}] = S^{-1}R[x]$, the result follows easily from Theorem 2.5. □

Corollary 2.8. *Let R be a semicommutative ring. Then:*
1) R is a weakArmendariz ring if and only if so is $R[x]$ (if and only if so is $R[x,x^{-1}]$).
2) R is a nilArmendariz ring if and only if so is $R[x]$ (if and only if so is $R[x,x^{-1}]$).
3) R is an Armendariz ring if and only if so is $R[x]$ (if and only if so is $R[x,x^{-1}]$).

Proof. 1) R is a weakArmendariz if and only if so is $R[x]$ by [15, Theorem 3.8]. On the other hand, $R[x]$ is a weakArmendariz if and only if so is $R[x,x^{-1}]$ by Corollary 2.8. Hence, we obtain the desired result.

2) Argue as 1).

3) R is an Armendariz ring if and only if so is $R[x]$ by [2, Theorem 2]. On the other hand, $R[x]$ is an Armendariz if and only if so is $R[x,x^{-1}]$ by Corollary 2.8. Hence, we obtain the desired result. □

Now, we will construct a wide class of rings satisfying the Armendariz-Like properties. For this, we study the transfer of this property to direct product of rings.

Theorem 2.9. *Let $(R_i)_{i=1,2,...,n}$ be a family of rings and let $R := \prod_{i=1}^n R_i$. Then:*
1) R is a weakArmendariz ring if and only if so is R_i for each $i = 1,\dots,n$.
2) R is a nilArmendariz ring if and only if so is R_i for each $i = 1,\dots,n$.
3) R is a semicommutative ring if and only if so is R_i for each $i = 1,\dots,n$.
4) R is an Armendariz ring if and only if so is R_i for each $i = 1,\dots,n$.
5) R is a reduced ring if and only if so is R_i for each $i = 1,\dots,n$.

Proof. We will prove the result for $i = 1,2$, and the Theorem will be established by induction on n.

1) Assume that $(R_1 \times R_2)$ is a weakArmendariz ring. We show that R_1 is a weakArmendariz ring (it is the same for R_2). Let $f = \sum_{i=0}^n a_i x^i$ and $g = \sum_{i=0}^m b_i x^i$ be two polynomials in $R_1[x]$ such that $fg = 0$, where n and m are positive integers. Set $f_1 = \sum_{i=0}^n (a_i,0)x^i$ and $g_1 = \sum_{i=0}^m (b_i,0)x^i \in (R_1 \times R_2)[x]$. We have $f_1 g_1 = (fg,0) = (0,0)$. Hence $C_{R_1\times R_2}(f_1)C_{R_1\times R_2}(g_1) \in nil(R_1 \times R_2)$ since $(R_1 \times R_2)$ is a weakArmendariz ring. But $C_{R_1\times R_2}(f_1)C_{R_1\times R_2}(g_1) = (C_{R_1}(f)C_{R_1}(g),0)$. Therefore, $C_{R_1}(f)C_{R_1}(g) \in nil(R_1)$ and this shows that R_1 is a weakArmendariz ring.

Conversely, assume that R_1 and R_2 are weakArmendariz rings. Let $f = \sum_{i=0}^n (a_i,e_i)x^i$ and $g = \sum_{i=0}^m (b_i,f_i)x^i \in (R_1 \times R_2)[x]$ such that $fg = 0$, where n and m are positive integers. Set $f_1 := \sum_{i=0}^n a_i x^i \in R_1[x]$, $f_2 := \sum_{i=0}^n e_i x^i \in R_2[x]$, $g_1 := \sum_{i=0}^m b_i x^i \in R_1[x]$ and $g_2 := \sum_{i=0}^m f_i x^i \in R_2[x]$. We have $0 = fg = (f_1 g_1, f_2 g_2)$ which implies that $f_1 g_1 = 0$ and $f_2 g_2 = 0$. Hence $C_{R_1}(f_1)C_{R_1}(g_1) \in nil(R_1)$ and $C_{R_2}(f_2)C_{R_2}(g_2) \in nil(R_2)$ since R_1 and R_2 are a weakArmendariz rings. But, $C_{R_1\times R_2}(f)C_{R_1\times R_2}(g) = (C_{R_1}(f_1)C_{R_1}(g_1), C_{R_2}(f_2)C_{R_2}(g_2))$. Therefore, $C_{R_1\times R_2}(f)C_{R_1\times R_2}(g) \in nil(R_1 \times R_2)$.

2) The same proved for nilArmendariz ring.

3) Assume that $(R_1 \times R_2)$ is a semicommutative ring. We show that R_1 is a semicommutative ring (it is the same for R_2). Let $a, b \in R_1$ such that $ab = 0$. We have $(a,0)(b,0) = (ab,0) = 0$ and so $(a,0)(x,y)(b,0) = 0$ for each $(x,y) \in R_1 \times R_2$ (since, $R_1 \times R_2$ is a semicommutative ring). Therefore, $axb = 0$ for each $x \in R_1$ and this shows that R_1 is a semicommutative ring.

Conversely, assume that R_1 and R_2 are a semicommutative rings. Let (x_1,y_1) and $(x_2,y_2) \in R_1 \times R_2$ such that $(x_1,y_1)(x_2,y_2) = 0$, which means that $x_1x_2 = 0$ and $y_1y_2 = 0$. Therefore, for each $a \in R_1$ and $b \in R_2$, $x_1ax_1 = 0$ and $y_1by_2 = 0$ (since R_1 and R_2 are a semicommutative rings), which means that $(x_1,y_1)(a,b)(x_2,y_2) = 0$, as desired.

4) By [4, Theorem 2.5].

5) Assume that $R_1 \times R_2$ is a reduced ring. We show that R_1 is a reduced ring (it is the same for R_2). Let $x \in R_1$ such that $x^n = 0$. We have $(x,0)^n = (x^n,0) = (0,0)$ and so $(x,0) = 0$ since $(R_1 \times R_2)$ is a reduced ring. Hence, R_1 is a reduced ring.

Conversely, assume that R_1 and R_2 are a reduced rings. We show that $(R_1 \times R_2)$ is a reduced ring. Let $(x,y) \in (R_1 \times R_2)$ such that $(x,y)^n = 0$. Hence, $(x,y)^n = (x^n,y^n) = 0$ which means that $x = 0$ and $y = 0$ since R_1 and R_2 are a reduced rings. Therefore, $(R_1 \times R_2)$ is a reduced ring as desired. □

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