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AN IRREDUCIBILITY CRITERION FOR THE SUM OF TWO RELATIVELY PRIME POLYNOMIALS

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Dedicated to the memory of Şerban Basarab

Abstract: We extend a result of Cavachi on sums of relatively prime polynomials by proving that a polynomial of the form $f(X)+p^k g(X)$, with f and g relatively prime polynomials with integer coefficients, $\deg f < \deg g$, and k a positive integer prime to $\deg g$ is irreducible over $\mathbb{Q}$ for all but finitely many prime numbers p.

Keywords: irreducible polynomials; prime numbers; resultant.

1. Introduction

If we add two algebraically relatively prime polynomials having coefficients in an arbitrary unique factorization domain, the resulting polynomial will not necessarily be irreducible, as one can easily check. However, if instead of the sum we consider linear combinations of the two polynomials, say $n_1 f(X) + n_2 g(X)$, then the resulting polynomials prove to be irreducible, provided some conditions on the factorization of n_1 and n_2 are satisfied. In this respect, several recent irreducibility criteria have been obtained for polynomials of the form $f(X)+pg(X)$, where f and g are relatively prime polynomials with rational coefficients, and p is a sufficiently large prime number. Inspired by some results of Fried [9] and Langmann [10], Cavachi [6] proved that for any relatively prime polynomials $f(X), g(X) \in \mathbb{Q}[X]$ with $\deg f < \deg g$, the polynomial $f(X) + pg(X)$ is irreducible over $\mathbb{Q}$ for all but finitely many prime numbers p. This result has been improved in [7] by providing an explicit lower bound b depending on f and g, such that for all primes $p > b$, the polynomial $f(X)+pg(X)$ is irreducible over $\mathbb{Q}$. The method in [7] was adapted in [4] in order to provide sharper bounds b as well as explicit upper bounds for the total number of factors over $\mathbb{Q}$ of linear combinations of the form $n_1 f(X)+n_2 g(X)$, where f and g are relatively prime polynomials with $\deg f \leqslant \deg g$, and n_1 and n_2 are non-zero integers with absolute value of n_2/n_1 exceeding an explicit lower

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bound. Similar results have been also provided for compositions of polynomials with integer coefficients [3] and for multiplicative convolutions of polynomials with integer coefficients [1], [2]. We obviously cannot replace the prime p in Cavachi's result by a sufficiently large positive integer n, since for instance a polynomial of the form $f(X)^2 - ng(X)^2$ with f and g relatively prime is obviously reducible whenever n is a square. However, given a pair of relatively prime polynomials f and g with $\deg f < \deg g$, some families of composite numbers n exist such that $f + ng$ is irreducible. In this respect, in [5] several irreducibility results have been provided for polynomials of the form $f(X)+p^k g(X)$ with f and g relatively prime polynomials with integer coefficients, $\deg f < \deg g$, p a prime number, and k a positive integer prime to $\deg g - \deg f$. The main result in [5], that partially relies on a Newton polygon argument, is the following extension of Cavachi's result.

Theorem A ([5, Theorem 1.1.]). *Let $f, g \in \mathbb{Z}[X]$ be two relatively prime polynomials with $\deg g = n$ and $\deg f = n - d$, $d \geqslant 1$. Then for any prime number p that divides none of the leading coefficients of f and g, and any positive integer k prime to d such that*

$$p^k \geqslant \left(2 + \frac{1}{2^{n+1-d} H(g)^{n+1}}\right)^{n+1-d} H(f) H(g)^n - \frac{H(f)}{H(g)},$$

the polynomial $f(X) + p^k g(X)$ is irreducible over $\mathbb{Q}$.

Here and henceforth for a polynomial $f(X) = a_0 + a_1 X + \cdots + a_n X^n \in \mathbb{Z}[X]$ of degree n, $H(f)$ stands for the usual height of f, that is

$$H(f) = \max\{|a_0|, |a_1|, ..., |a_n|\}.$$

In this paper we will complement the results in [6], [4], [5] and [7] by proving that the result of Cavachi also holds if we replace the prime p by a prime power p^k with k prime to $\deg g$. We will actually prove the following effective result that provides an explicit lower bound for p depending on f and g, that once exceeded, will ensure the irreducibility of $f(X) + p^k g(X)$ over $\mathbb{Q}$.

Theorem 1.1. *Let $f, g \in \mathbb{Z}[X]$ be relatively prime polynomials with $\deg f = m$, $\deg g = n$, and $m < n$. Then for any prime number p and any positive integer k prime to n such that*

$$p > \left(2 + \frac{1}{2^{k(m+1)(n-1)}}\right)^{(m+1)(n-1)} H(f)^{n-1} H(g)^{m(n-1)+1},$$

the polynomial $f(X) + p^k g(X)$ is irreducible over $\mathbb{Q}$.

In particular, we have the following corollary.

Corollary 1.2. *Let $f, g \in \mathbb{Z}[X]$ be two relatively prime polynomials with $\deg f < \deg g$, and let k be a positive integer prime to $\deg g$. Then the polynomial $f(X) + p^k g(X)$ is irreducible over $\mathbb{Q}$ for all but finitely many prime numbers p.*

We note that Theorem 1.1 also holds in the particular case $\deg f = 0$, that is when f is a nonzero constant polynomial, say $f = a \in \mathbb{Z} \setminus \{0\}$. Actually, in this case one may prove a better result, namely that the irreducibility of $a + p^k g$ will hold provided a and the leading coefficient of g are not divisible by p, so here we do not need to ask p to exceed a certain lower bound, as in the statement of Theorem 1.1. More precisely, in this case we have the following result.

Theorem 1.3. *Let $g \in \mathbb{Z}[X]$ be a polynomial with $\deg g = n$ and leading coefficient b_n, and let a be a nonzero integer. Then for any prime number p that does not divide ab_n, and any positive integer k prime to n, the polynomial $a + p^k g(X)$ is irreducible over $\mathbb{Q}$.*

This result is also a special case of Theorem A (the case $d = n$ that was discussed in [5] in Remark 2.1), and its proof follows immediately by using the following celebrated irreducibility criterion of Dumas [8].

Irreducibility criterion of Dumas. *Let $f(X) = a_0 + a_1 X + \cdots + a_n X^n$ be a polynomial with integer coefficients, and let p be a prime number. If*

i) $\frac{\nu_p(a_i)}{i} > \frac{\nu_p(a_n)}{n}$ *for* $i = 1, \ldots, n-1$,
ii) $\nu_p(a_0) = 0$,
iii) $\gcd(\nu_p(a_n), n) = 1$,

then f is irreducible over $\mathbb{Q}$.

Here for an integer n and a prime number p, $\nu_p(n)$ stands for the largest integer i such that $p^i \mid n$ (by convention, $\nu_p(0) = \infty$).

One may easily obtain sharper results, if some additional information on the coefficients or on the roots of f and g is available. In this respect, we will also prove the following irreducibility criterion.

Theorem 1.4. *Let $f(X) = \sum_{i=0}^{m} a_i X^i, g(X) = \sum_{i=0}^{n} b_i X^i \in \mathbb{Z}[X]$ be two relatively prime polynomials with $a_m b_n \neq 0$, $m < n$, and assume that $|b_n| > |b_0| + \cdots + |b_{n-1}|$. Then for any prime number $p > |b_n|^n (|a_0| + \cdots + |a_m|)^{n-1}$ and any positive integer k prime to n, the polynomial $f(X) + p^k g(X)$ is irreducible over $\mathbb{Q}$.*

In particular, one obtains the following result.

Corollary 1.5. *Let $f(X) = \sum_{i=0}^{m} a_i X^i, g(X) = \sum_{i=0}^{n} b_i X^i \in \mathbb{Z}[X]$ with $a_m b_n \neq 0$, $m < n$, and assume that $|a_0| \geqslant |a_1| + \cdots + |a_m|$ and $|b_n| > |b_0| + \cdots + |b_{n-1}|$. Then for any prime number $p > |b_n|^n (|a_0| + \cdots + |a_m|)^{n-1}$ and any positive integer k prime to n, the polynomial $f(X) + p^k g(X)$ is irreducible over $\mathbb{Q}$.*

The proof of Theorem 1.1 will not rely on a Newton polygon argument, that was crucial in the proof of Theorem A. Here the proof will rely on a simultaneous analysis of some resultants associated to the alleged factors of $f(X) + p^k g(X)$. We end this section by noting that the lower bound on the prime p in the statement of Theorem 1.1 may be replaced by

$$\left(2 + \frac{1}{2^{(m+1)(n-1)}}\right)^{(m+1)(n-1)} H(f)^{n-1} H(g)^{m(n-1)+1}.$$

which is independent on k.

2. Proof of the main results

Proof of Theorem 1.1. For the Proof of Theorem 1.1 we will adapt some of the ideas in [4], [5] and [7]. We will actually prove a sharper result, by showing that for $m \geqslant 1$ the same conclusion on the irreducibility of $f+p^k g$ holds if we replace the condition on p in the statement of Theorem 1.1 by

$$p \geqslant \left(2+\frac{1}{2^{k(m+1)(n-1)}A}\right)^{(m+1)(n-1)} H(f)^{n-1}H(g)^{m(n-1)+1} \tag{1}$$

with $A = H(f)^{k(n-1)-1}H(g)^{k(m(n-1)+1)+1} \geqslant 1$.

So let $f(X) = a_0 + a_1X + \cdots + a_mX^m$ and $g(X) = b_0 + b_1X + \cdots + b_nX^n$ be two relatively prime polynomials with integer coefficients, $a_m b_n \neq 0$, $1 \leqslant m < n$, and let p be a prime number and k a positive integer prime to n satisfying (1). Now let us assume to the contrary that $f(X)+p^k g(X)$ is reducible, that is

$$f(X)+p^k g(X) = f_1(X)f_2(X), \tag{2}$$

with $f_1(X), f_2(X) \in \mathbb{Z}[X]$ and $\deg f_1 \geqslant 1$, $\deg f_2 \geqslant 1$, say

$$f_1(X) = c_0 + c_1X + \cdots + c_sX^s,$$
$$f_2(X) = d_0 + d_1X + \cdots + d_tX^t,$$

$c_0, \ldots c_s, d_0, \ldots d_t \in \mathbb{Z}$, $c_s d_t \neq 0$, and $s \geqslant 1$, $t \geqslant 1$, $s+t = n$. By equating the coefficients in (2) we see that $p^k b_n = c_s d_t$. Let us denote $c_s = p^\alpha c'_s$ and $d_t = p^\beta d'_t$, with $\alpha, \beta \in \mathbb{N}$, $c'_s, d'_t \in \mathbb{Z}$ and $p \nmid c'_s d'_t$. In view of (1) we deduce that $p \nmid b_n$, so we have $\alpha+\beta = k$.

Now we are going to estimate the resultants $R(g, f_1)$ and $R(g, f_2)$. Since g and $f_1 f_2$ are relatively prime polynomials, both $R(g, f_1)$ and $R(g, f_2)$ must be non-zero integer numbers, so in particular we have

$$|R(g, f_1)| \geqslant 1 \qquad \text{and} \qquad |R(g, f_2)| \geqslant 1. \tag{3}$$

If we decompose f_1 and f_2, say

$$f_1(X) = c_s(X-\theta_1)\cdots(X-\theta_s),$$
$$f_2(X) = d_t(X-\xi_1)\cdots(X-\xi_t),$$

with $\theta_1, \ldots, \theta_s, \xi_1, \ldots, \xi_t \in \mathbb{C}$, then

$$|R(g, f_1)| = |c_s|^n \prod_{1\leqslant j\leqslant s} |g(\theta_j)| \qquad \text{and} \qquad |R(g, f_2)| = |d_t|^n \prod_{1\leqslant j\leqslant t} |g(\xi_j)|\,. \tag{4}$$

Since the roots θ_j of f_1 and the roots ξ_j of f_2 are also roots of $f(X)+p^k g(X)$, we have

$$g(\theta_j) = -\frac{f(\theta_j)}{p^k} \qquad \text{and} \qquad g(\xi_j) = -\frac{f(\xi_j)}{p^k} \tag{5}$$

and moreover, since f and g are relatively prime, $f(\theta_j) \neq 0$ and $g(\theta_j) \neq 0$ for any index $j \in \{1, \ldots, s\}$, and also $f(\xi_j) \neq 0$ and $g(\xi_j) \neq 0$ for any index $j \in \{1, \ldots, t\}$. Using now (4) and (5), we obtain

$$|R(g, f_1)| = \frac{p^{n\alpha}|c_s'|^n}{p^{ks}} \prod_{1 \leqslant j \leqslant s} |f(\theta_j)| \qquad \text{and} \qquad |R(g, f_2)| = \frac{p^{n\beta}|d_t'|^n}{p^{kt}} \prod_{1 \leqslant j \leqslant t} |f(\xi_j)| \,. \tag{6}$$

We will prove now that we either have $ks > n\alpha$, or $kt > n\beta$. To prove this we first note that $ks - n\alpha + kt - n\beta = k(s+t) - n(\alpha + \beta) = kn - nk = 0$. This shows that it is sufficient to prove that none of the integers $ks - n\alpha$ and $kt - n\beta$ can actually vanish. Indeed, if we assume that $ks = n\alpha$, say, then we must also have $kt = n\beta$, and since k is prime to n, we deduce that k must divide both α and β. On the other hand, since $\alpha + \beta = k$ and $\alpha \geqslant 0$, $\beta \geqslant 0$, we deduce that one of α and β must be equal to 0, while the other one must be equal to k, say $\alpha = 0$ and $\beta = k$. In particular, this yields $ks = 0$, which obviously cannot hold, so we must either have $ks > n\alpha$, or $kt > n\beta$. Without loss of generality, let us assume that $ks > n\alpha$ and hence $ks - n\alpha \geqslant 1$. Therefore, in view of (6) we deduce that

$$|R(g, f_1)| \leqslant \frac{|c_s'|^n}{p} \prod_{1 \leqslant j \leqslant s} |f(\theta_j)| \leqslant \frac{|b_n|^n}{p} \prod_{1 \leqslant j \leqslant s} |f(\theta_j)| \,. \tag{7}$$

We now proceed to find an upper bound for $|f(\theta_j)|$. The equality $f(\theta_j) + p^k g(\theta_j) = 0$ implies

$$(a_0 + p^k b_0) + \cdots + (a_m + p^k b_m)\theta_j^m + p^k b_{m+1}\theta_j^{m+1} + \cdots + p^k b_n \theta_j^n = 0,$$

from which we deduce that

$$\begin{aligned} p^k|b_n| \cdot |\theta_j|^n &\leqslant |a_0| + p^k|b_0| + (|a_1| + p^k|b_1|) \cdot |\theta_j| + \cdots + (|a_m| + p^k|b_m|) \cdot |\theta_j|^m \\ &\quad + p^k|b_{m+1}| \cdot |\theta_j|^{m+1} + \cdots + p^k|b_{n-1}| \cdot |\theta_j|^{n-1} \\ &\leqslant \left(H(f) + p^k H(g)\right)\left(1 + |\theta_j| + \cdots + |\theta_j|^{n-1}\right). \end{aligned}$$

Therefore, either $|\theta_j| \leqslant 1$, or if not, we find

$$p^k|b_n| \cdot |\theta_j|^n < \left(H(f) + p^k H(g)\right) \cdot \frac{|\theta_j|^n}{|\theta_j| - 1},$$

so in both cases we have

$$|\theta_j| < 1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right). \tag{8}$$

Now, since obviously

$$|f(\theta_j)| \leqslant H(f) \cdot (1 + |\theta_j| + \cdots + |\theta_j|^m)\,,$$

inequality (8) yields

$$|f(\theta_j)| < H(f) \cdot \frac{\left[1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right]^{m+1}}{\frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)}. \tag{9}$$

Using now (7) and (9), we obtain

$$|R(g, f_1)| < \frac{|b_n|^n}{p} \left[|b_n| H(f) \cdot \frac{\left[1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right]^{m+1}}{\frac{H(f)}{p^k} + H(g)}\right]^s.$$

Since $s \leqslant n - 1$, all we need to prove is that our assumption on p will force

$$\frac{|b_n|^{\frac{n}{n-1}}}{p^{\frac{1}{n-1}}} \left[|b_n| H(f) \cdot \frac{\left[1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right]^{m+1}}{\frac{H(f)}{p^k} + H(g)}\right] \leqslant 1,$$

that is

$$|b_n|^{1+\frac{n}{n-1}} \left[1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right]^{m+1} \leqslant \frac{p^{\frac{1}{n-1}}}{p^k} + \frac{p^{\frac{1}{n-1}} H(g)}{H(f)},$$

which is equivalent to

$$\frac{|b_n|^{1+\frac{n}{n-1}}}{|b_n|^{1+m}} \left[|b_n| + \frac{H(f)}{p^k} + H(g)\right]^{m+1} \leqslant \frac{p^{\frac{1}{n-1}}}{p^k} + \frac{p^{\frac{1}{n-1}} H(g)}{H(f)}.$$

Now, since for $n \geqslant 2$ and $m \geqslant 1$ we have

$$\frac{|b_n|^{1+\frac{n}{n-1}}}{|b_n|^{1+m}} \leqslant \frac{|b_n|^{1+\frac{n}{n-1}}}{|b_n|^2} = |b_n|^{\frac{1}{n-1}},$$

it will be sufficient to prove that

$$|b_n|^{\frac{1}{n-1}} \left[|b_n| + \frac{H(f)}{p^k} + H(g)\right]^{m+1} \leqslant \frac{p^{\frac{1}{n-1}} H(g)}{H(f)},$$

that is

$$p \geqslant \frac{H(f)^{n-1}}{H(g)^{n-1}} \cdot |b_n| \cdot \left[|b_n| + \frac{H(f)}{p^k} + H(g)\right]^{(m+1)(n-1)}.$$

Now, since $|b_n| \leqslant H(g)$, it suffices to prove that

$$p \geqslant \frac{H(f)^{n-1}}{H(g)^{n-1}} \cdot H(g) \cdot \left[2H(g) + \frac{H(f)}{p^k}\right]^{(m+1)(n-1)},$$

or equivalently, that

$$p \geqslant H(f)^{n-1} H(g)^{m(n-1)+1} \cdot \left[2 + \frac{H(f)}{p^k H(g)}\right]^{(m+1)(n-1)}. \tag{10}$$

Using the idea in [5], if we define now the function

$$\mathcal{F}(x) := H(f)^{n-1} H(g)^{m(n-1)+1} \cdot \left[2 + \frac{H(f)}{x H(g)}\right]^{(m+1)(n-1)} \qquad \text{for} \quad x > 0,$$

then in view of (10) we have to search for a value of p as small as possible such that $p \geqslant \mathcal{F}(p^k)$. In this respect, since $\mathcal{F}$ is a decreasing function, it will be sufficient to search for a suitable $\delta > 0$, such that

$$p \geqslant B := \delta H(f)^{n-1} H(g)^{m(n-1)+1}$$

and

$$B \geqslant \mathcal{F}\left(B^k\right).$$

Therefore it will be sufficient to find a δ as small as possible satisfying

$$\delta \geqslant \left(2 + \frac{1}{\delta^k H(f)^{k(n-1)-1} H(g)^{k(m(n-1)+1)+1}}\right)^{(m+1)(n-1)},$$

that is

$$\delta \geqslant \left(2 + \frac{1}{\delta^k A}\right)^{(m+1)(n-1)}, \tag{11}$$

recalling our notation $A = H(f)^{k(n-1)-1} H(g)^{k(m(n-1)+1)+1}$. A suitable candidate for a δ satisfying (11) is easily seen to be

$$\delta_0 := \left(2 + \frac{1}{2^{k(m+1)(n-1)} A}\right)^{(m+1)(n-1)},$$

since obviously $\delta_0 > 2^{(m+1)(n-1)}$. This proves that for

$$p \geqslant \left(2 + \frac{1}{2^{k(m+1)(n-1)} A}\right)^{(m+1)(n-1)} H(f)^{n-1} H(g)^{m(n-1)+1}$$

we have $|R(g, f_1)| < 1$, which contradicts (3), and completes the proof. ■

Proof of Theorem 1.4. We will use the notation from the proof of Theorem 1.1. The case $m = 0$ follows directly from Theorem 1.3, so we may assume that $m \geqslant 1$ and hence $n \geqslant 2$. Therefore, our assumption that $p > |b_n|^n (|a_0| + \cdots + |a_m|)^{n-1}$ shows that $p > |a_0| + \cdots + |a_m|$ and therefore

$$p^k > |a_0| + \cdots + |a_m|. \tag{12}$$

On the other hand, the fact that $|b_n| > |b_0| + \cdots + |b_{n-1}|$ implies $|b_n| \geqslant 1 + |b_0| + \cdots + |b_{n-1}|$, so in view of (12) we deduce that

$$p^k|b_n| \geqslant p^k + \sum_{i=0}^{n-1} p^k|b_i| > \sum_{i=0}^{m} |a_i| + \sum_{i=0}^{n-1} p^k|b_i| \geqslant \sum_{i=0}^{m} |a_i + p^k b_i| + \sum_{i=m+1}^{n-1} p^k|b_i|, \quad (13)$$

with the rightmost sum in (13) appearing only if $n - m \geqslant 2$. In view of (13) we deduce that all the roots θ of $f(X) + p^k g(X)$ satisfy $|\theta| \leqslant 1$, so by (7) we obtain

$$|R(g, f_1)| \leqslant \frac{|b_n|^n(|a_0| + \cdots + |a_m|)^s}{p} \leqslant \frac{|b_n|^n(|a_0| + \cdots + |a_m|)^{n-1}}{p}.$$

Therefore, if $p > |b_n|^n(|a_0| + \cdots + |a_m|)^{n-1}$, then $f + p^k g$ must be irreducible over $\mathbb{Q}$. ∎

Proof of Corollary 1.5. Here too we may assume $m \geqslant 1$. The fact that $|a_0| \geqslant |a_1| + \cdots + |a_m|$ forces the roots of f to satisfy $|z| \geqslant 1$, while condition $|b_n| > |b_0| + \cdots + |b_{n-1}|$ shows that the roots of g must satisfy $|z| < 1$. Therefore f and g must be algebraically relatively prime, and one applies Theorem 1.4. ∎

Remark 2.1. We end by noting that slightly sharper conditions than those exhibited in Theorem 1.1 may be also obtained when $g(0) \neq 0$ and f is a monomial, say $f(X) = a_m X^m$ for some $m \in \{1, \ldots, n-1\}$, $a_m \neq 0$. In this case $H(f) = a_m$ and instead of (9) we obtain

$$|f(\theta_j)| < H(f)\left(1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right)^m,$$

and therefore

$$|R(g, f_1)| < \frac{|b_n|^n}{p} H(f)^{n-1}\left(1 + \frac{1}{|b_n|} \cdot \left(\frac{H(f)}{p^k} + H(g)\right)\right)^{m(n-1)}.$$

The reader may easily check that the same conclusion on the irreducibility of $f + p^k g$ holds in this case for primes p satisfying

$$p > \left(2 + \frac{1}{2^{km(n-1)}}\right)^{m(n-1)} H(f)^{n-1} H(g)^{m(n-1)+1}.$$

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