SUT Journal of Mathematics
Vol. 39, No. 2 (2003), 161–170

f-pluriharmonic maps on manifolds with f-structures

Cornelia Livia Bejan and Michèle Benyounes

(Received October 1, 2003)

Abstract. We introduce and study here the notion of f-pluriharmonicity, as the extension of pluriharmonicity from the context of the almost Hermitian manifolds, to the manifolds endowed with f-structures, which are defined by K. Yano in [12]. Then we relate f-pluriharmonicity with $\pm f$-holomorphicity and f-(1,1)–geodesicity. We generalize a result obtained by S. Udagawa in [11], we give some applications by using the complex sectional curvature defined by T. Siu in [9] and we construct some examples.

AMS 2000 *Mathematics Subject Classification.* 53C15.

Key words and phrases. f-structures on manifolds, pluriharmonic maps.

Introduction

A basic result (of Lichnerowicz type), relating the holomorphicity with the harmonicity, states that if M and N are almost Hermitian manifolds such that M is cosymplectic and N is (1,2)-symplectic, then any $\pm$ holomorphic map $\Phi : M \to N$ is harmonic, [4]. If M and N are Riemann surfaces, then any $\pm$ holomorphic map $\Phi : M \to N$ is harmonic with respect to any Hermitian metrics. The notion of pluriharmonic map is a natural extension of harmonic map from a Riemann surface. As it is pointed out in [5], a Hermitian manifold is cosymplectic (resp. Kähler) iff any pluriharmonic map from M is harmonic (resp. (1,1)–geodesic).

It is well known that if $\Phi : M \to N$ is a $\pm$ holomorphic (resp. pluriharmonic) map between Kähler manifolds, then Φ is pluriharmonic (resp. harmonic), [11]. An interesting problem is the converse: find sufficient conditions under which any pluriharmonic map between Kähler manifolds is $\pm$ holomorphic. Some results on this problem due to Dajczer-Gromoll, Dajczer-Rodriguez, Dajczer-Thorbergsson, are extended by S. Udagawa (see [11] and the references therein).

In the present paper we deal with the same problem in the framework of the f-structures, which generalize the almost Hermitian structure [12]. We recall about this notion in section 1, where we provide some basic facts. In section 2, as an extension from the almost Hermitian context, we introduce the f-pluriharmonicity and f-(1,1)–geodesicity between manifolds endowed with f-strutures. Concerning the above problem, we give here some necessary and sufficient conditions under which an f-pluriharmonic map of rank ≥ 3 from a manifold endowed with an f-struture into a complex space form $N\,(c)$ $(c \neq 0)$ is $\pm f$-holomorphic. This theorem generalizes Udagawa's result [11, Theorem 1], which is proved by a different method. We also give an example and a consequence. In section 3 we deal with the complex sectional curvature which was defined by T. Siu in [9]. At the end, we obtain some results on pluriharmonic maps, corresponding to Sampson's theorem on harmonic maps.

§1. f-structures

The notion of f-structure was introduced in [12] as a natural extension of the almost Hermitian stucture to the manifolds of not necessarily even dimension. A rich literature is devoted to the subject, from which we point out only some authors: K. Yano and M. Kon, J. K. Rawnsley, D. Blair, A. Bejancu, S. Ianus, F. E. Burstall, etc.

A manifold M carries an f-*structure* F if:

(1.1) $$F \in C^{\infty}(\mathrm{End}(TM)),\ \mathrm{rank}\, f = \text{constant},\ F^3 + F = 0.$$

There exists always a Riemannian structure g with respect to which F is skew-symmetric:

(1.2) $$g(FX, Y) + g(X, FY) = 0, \quad \forall X, Y \in C^{\infty}(TM).$$

A couple (g, F) is called a *Riemannian* f-*structure*. In particular, it can be almost Hermitian, almost contact [2], etc.

Remark. The complexified tangent bundle $T^{\mathbb{C}}M = TM \otimes \mathbb{C}$ splits into a direct sum, corresponding to the eigenvalues $i, -i, 0$ of the complexification of F:

(1.3) $$T^{\mathbb{C}}M = T^{+}M \oplus T^{-}M \perp T^{0}M,$$

where $\oplus$ and $\perp$ denote the direct and the orthogonal sum respectively. Then, $T^{+}M = \overline{T^{-}M}$ and $T^{0}M = \mathrm{Ker}F \otimes \mathbb{C}$.

Three basic notions concerning f-structures will be taken into account:

I. An f-stucture F is *integrable* if $T^{+}M$ is closed under the Lie bracket.

Example 1.1. A CR-manifold is a $(2n+1)$-dimensional manifold M carrying a rank n complex subbundle V of $T^{\mathbb{C}}M$ such that:

$$V \cap \overline{V} = 0 \text{ and } [\mathbb{C}^\infty(V), \mathbb{C}^\infty(V)] \subset \mathbb{C}^\infty(V).$$

The Levi distribution of M is the rank $2n$ real subbundle of $T^{\mathbb{C}}M$ given by $H = \mathrm{Re}(V \oplus \overline{V})$, which carries the complex structure:

$$\mathcal{J} : H \longrightarrow H\,,\ \mathcal{J}(Z + \overline{Z}) = i(Z - \overline{Z})\,,\ Z \in V.$$

There always exists a 1-dimensional distribution K such that $TM = H \oplus K$. Then

$$F : TM \longrightarrow TM \text{ defined by } F(X + \xi) = \mathcal{J}X,\ \forall X \in \mathbb{C}^\infty(H), \xi \in \mathbb{C}^\infty(K),$$

is an integrable f-structure.

Example 1.2. A CR-submanifold M of an almost Hermitian manifold $(N, g, \mathcal{J})$ is defined as carrying an invariant distribution D (i.e. $\mathcal{J}(D) = D$) whose orthogonal complement $D^\perp$(i.e. $TM = D \perp D^\perp$) is anti-invariant ($\mathcal{J}(D^\perp) \subset TM^\perp$), [1]. If we define:

(1.4)
$$F : TM \longrightarrow TM, \quad F(X + \xi) = \mathcal{J}X\,,\ \forall X \in \mathbb{C}^\infty(D), \xi \in \mathbb{C}^\infty(D^\perp),$$

then (g, F) is a Riemannian f-structure which is integrable when $(N, g, \mathcal{J})$ is Hermitian. In particular, the integrable f-structure on the sphere $S^{2n-1} \subset \mathbb{C}^n$ was noticed in [3].

II. We say that a Riemannian f-stucture (g, F) on a manifold M satisfies *the condition* $\tilde{A}$ if:

$$(\tilde{A}) \qquad \begin{cases} \nabla_Z \overline{W} \in \mathbb{C}^\infty(T^- M)\,,\ \forall Z, W \in \mathbb{C}^\infty(T^+ M) \text{ and} \\ \nabla_\xi \xi \in \mathbb{C}^\infty \mathrm{Ker}(F)\,,\ \forall \xi \in \mathbb{C}^\infty \mathrm{Ker}(F), \end{cases}$$

where ∇ is the Levi-Civita connection of g.

Remark. In the literature, the first condition of $\tilde{A}$ is called condition A, [3]. When the distribution Ker$F$ is parallel with respect to ∇, then A and $\tilde{A}$ coincide.

Example 1.3. Let $(N, g, \mathcal{J})$ be an almost Hermitian $(1, 2)$-symplectic manifold and let (K, G) be a Riemannian manifold. Then, the Riemannian product manifold $(N \times K, g \oplus G)$ carries the Riemannian f-structure $F = \mathcal{J} \oplus 0$, which satisfies condition $\tilde{A}$.

Proposition 1.4. *Any totally geodesic CR-submanifold of a Kähler manifold carries a Riemannian f-structure which satisfies condition $\tilde{A}$ and has parallel kernel.*

Proof. Let M be a totally geodesic CR-submanifold of a Kähler manifold $(N, g, \mathcal{J})$ and let F be the f-structure on M defined by (1.4). Then, $\mathrm{Ker}F = D^{\perp}$ is parallel. That is any $\xi \in \mathbb{C}^{\infty}(\mathrm{Ker}F)$, $X \in \mathbb{C}^{\infty}(TM)$ satisfy ${}^{M}\nabla_X \xi \in \mathbb{C}^{\infty}(\mathrm{Ker}F)$ or, equivalently, $\mathcal{J}({}^{M}\nabla_X \xi)$ is orthogonal to TM, since from Gauss formula, we have:

$$g(\mathcal{J}^{M}\nabla_X \xi, Y) = g(\mathcal{J}^{N}\nabla_X \xi, Y) = g({}^{N}\nabla_X \mathcal{J}\xi, Y) = -g(\mathcal{J}\xi, {}^{N}\nabla_X Y) =$$

$$-g(\mathcal{J}\xi, {}^{M}\nabla_X Y) = 0\,,\ \forall Y \in \mathbb{C}^{\infty}(TM),$$

where ${}^{M}\nabla$ and ${}^{N}\nabla$ denote the Levi-Civita connections on M and N respectively. The condition A is satisfied since N is Kähler and M totally geodesic. From the above remark, A and $\tilde{A}$ coincide. □

In particular, it follows:

Corollary 1.5. *Any totally geodesic hypersurface of a Kähler manifold carries a Riemannian f-structure which satisfies condition $\tilde{A}$ and has parallel kernel.*

The condition of totally geodesicity can not be removed from Proposition 1.4 and Corollary 1.5 since the f-structure on the sphere $S^{2n-1} \subset \mathbb{C}^n$ noticed in [3] does not even satisfy condition A.

III. Any map $\Phi : (M, g^M, F^M) \to (N, g^N, F^N)$ between manifolds with Riemannian f-structures is *f-holomorphic* if:

$$d\Phi \circ F^M = F^N \circ d\Phi \tag{1.5}$$

Equivalently, we have:

$$d\Phi(T^+M) \subset T^+N\,,\ d\Phi(T^-M) \subset T^-N\,,\ d\Phi(T^0M) \subset T^0N.$$

A similar definition can be given for f-antiholomorphic. We say that Φ is *$\pm f$-holomorphic* if it is f-holomorphic or f-antiholomorphic.

§2. f-pluriharmonic maps

Let $\Phi : (M, g, F) \longrightarrow (N, G)$ be a map from a manifold with a Riemannian f-structure to a Riemannian manifold and let $h = \nabla d\Phi$ denote its second fundamental form.
The notions of $(1,1)$–geodesic map [4] and pluriharmonic map [10], [11] can be extended from the almost Hermitian case to the case of f-structures, as follows:

Definition 2.1. (i) Φ *is* f-$(1,1)$*–geodesic if:*

$$h(X, Y) + h(FX, FY) = 0\,,\ \forall X, Y \in \mathbb{C}^\infty(TM) \tag{2.1}$$

(ii) Φ *is* f*-pluriharmonic if:*

$$h(X, \xi) = 0\,,\ \forall X \in \mathbb{C}^\infty(TM)\,,\ \forall \xi \in \mathbb{C}^\infty(\mathrm{Ker}F) \tag{2.2}$$

and

$$\nabla^{1,0}\overline{\partial}\Phi = {}^{\Phi}\nabla_Z d\text{"}\Phi(\overline{W}) - d\text{"}\Phi({}^{-}\nabla_Z \overline{W}) = 0\,,\ \forall Z, W \in \mathbb{C}^\infty(T^+M) \tag{2.3}$$

where ${}^{-}\nabla_Z \overline{W}$ *is the projection of* ${}^{M}\nabla_Z \overline{W}$ *on* T^-M *and* $d\text{"}\Phi = d\Phi_{|T^-M}$.

Remark. Any f-$(1,1)$–geodesic map is harmonic.

By a straightforward computation, we obtain:

Lemma 2.2. *The map* Φ *is* f-$(1,1)$*–geodesic if and only if it satisfies* (2.2) *and its restriction to any complex curve is harmonic.*

We remark that Φ restricted to any complex curve is harmonic if and only if:

$$h(Z, \overline{W}) = 0\,,\ \forall Z, W \in \mathbb{C}^\infty(T^+M). \tag{2.4}$$

Proposition 2.3. *Any two of the following conditions imply the other one:*

(i) Φ *is* f-$(1,1)$*–geodesic ;*

(ii) Φ *is* f*-pluriharmonic;*

(iii) Φ *satisfies* (2.2) *and*

$$^{\oplus}\nabla_Z \overline{W} \in \mathrm{Ker}(d\Phi)\,,\ \forall Z, W \in \mathbb{C}^\infty(T^+M), \tag{2.5}$$

where ${}^{\oplus}\nabla_Z \overline{W}$ *denotes the projection of* ${}^{M}\nabla_Z \overline{W}$ *on* $T^+M \perp T^0M$.

Proof. For any $Z, W \in \mathbb{C}^\infty(T^+M)$, we have:

$$\begin{aligned}\nabla^{1,0}\overline{\partial}\Phi(Z,W) = &\ {}^{\Phi}\nabla_Z d"\Phi(\overline{W}) - d"\Phi({}^{-}\nabla_Z\overline{W})\\ = &\ {}^{\Phi}\nabla_Z d\Phi(\overline{W}) - d\Phi({}^{M}\nabla_Z\overline{W}) + d\Phi({}^{\oplus}\nabla_Z\overline{W})\\ = &\ h(Z,\overline{W}) + d\Phi({}^{\oplus}\nabla_Z\overline{W}).\end{aligned}$$

And the statement follows from Lemma 2.2. □

Remarks. We have:

1. F satisfies condition A if and only if ${}^{\oplus}\nabla_Z\overline{W} = 0$, $\forall Z, W \in \mathbb{C}^\infty(T^+M)$.
2. If F satisfies condition A, then the notion of f-(1,1)–geodesic coincides with f-pluriharmonic.
3. If (M, g, F) is almost Hermitian $(1,2)$-symplectic (in particular Kähler), then (2.1) coincides with the pluriharmonicity considered in [11].
4. If (M, g, F) is almost Hermitian, we obtain the following:
   - (i) Any pluriharmonic map is harmonic if and only if M is cosymplectic. This statement was obtained in the Hermitian case in [5].
   - (ii) Any pluriharmonic map is $(1,1)$–geodesic if and only if M is $(1,2)$-symplectic. In the Hermitian case, Ohnita-Vali proved in [5] that any pluriharmonic map is $(1,1)$–geodesic if and only if M is Kähler.

Theorem 2.4. *Let (M, g, F) be a manifold with Riemannian integrable f-structure satisfying condition $\tilde{A}$. Then any f-pluriharmonic map $\Phi : M \longrightarrow N(c)$ of rank $(d\Phi\circ F) \geq 3$ into a complex space form ($c \neq 0$) is $\pm f$-holomorphic if and only if:*

$$R^M(Z,W)\overline{W} \in \mathrm{Ker}(d\Phi)\,,\ \forall Z, W \in \mathbb{C}^\infty(T^+M)\ or\, W \in \mathbb{C}^\infty(KerF). \tag{2.6}$$

Remark. In particular, if M is Kähler, then (2.6) is automatically satisfied and the theorem was obtained in [11] by a slightly different method.

Lemma 2.5. *If (M, g, F) is a manifold with Riemannian integrable f-structure satisfying condition $\tilde{A}$ and $\Phi : M \longrightarrow N$ is an f-pluriharmonic map into a Riemannian manifold, then* (2.6) *is equivalent to:*

$$R^N(S,Q)\overline{Q} = 0\,, \tag{2.7}$$

where $S = d\Phi(Z)$, $Q = d\Phi(W)$, $\forall Z, W \in \mathbb{C}^\infty(T^+M)$ or $W \in \mathbb{C}^\infty(\mathrm{Ker}F)$.

Proof. From the assumptions, we have

$$
\begin{aligned}
R^N(S,Q)\overline{Q} &= R^N(d\Phi(Z), d\Phi(W))d\Phi(\overline{W}) \\
&= \nabla_{d\Phi(Z)}\nabla_{d\Phi(W)}d\Phi(\overline{W}) - \nabla_{d\Phi(W)}\nabla_{d\Phi(Z)}d\Phi(\overline{W}) \\
&\quad - \nabla_{d\Phi[Z,W]}d\Phi(\overline{W}) \\
&\text{(from } f\text{-pluriharmonicity)} \\
&= \nabla_{d\Phi(Z)}d\Phi(\nabla_W\overline{W}) - \nabla_{d\Phi(W)}d\Phi(\nabla_Z\overline{W}) - \nabla_{d\Phi[Z,W]}d\Phi(\overline{W}) \\
&= d\Phi(R^M(Z,W)\overline{W}),
\end{aligned}
$$

where we used the integrability of F (when $W \in \mathbb{C}^\infty(KerF)$), the condition $\tilde{A}$ and again pluriharmonicity. □

Proof of Theorem 2.4. If Φ is $\pm f$-holomorphic, then (2.6) follows as being equivalent with (2.7), which is satisfied since N is Kähler. Conversely, let assume (2.6) or equivalently (2.7). For any $P \in \mathbb{C}^\infty(T^{\mathbb{C}}N)$, let P' and P'' denote its holomorphic and antiholomorphic part, respectively. Then,

$$\overline{P''} = \overline{P}' \quad \text{and} \quad \overline{P'} = \overline{P}'' \tag{2.8}$$

Step 1. We prove that Φ satisfies either (2.9) or (2.10), where :

$$d\Phi(T^+M) \subset T^{1,0}N\,,\ d\Phi(T^-M) \subset T^{0,1}N; \tag{2.9}$$

$$d\Phi(T^+M) \subset T^{0,1}N\,,\ d\Phi(T^-M) \subset T^{1,0}N. \tag{2.10}$$

Suppose that both (2.9) and (2.10) don't hold. Then there exists $S = d\Phi(Z)$, $Z \in \mathbb{C}^\infty(T^+M)$, such that $S', S'' \neq 0$ (hence $\mathbb{R}$-lineareły independent). Since $\dim_{\mathbb{R}} d\Phi(T^+M) = \mathrm{rank}(d\Phi \circ F) \geq 3$, there exists $Y \in \mathbb{C}^\infty(T^+M)$ such that either $S', S'', [d\Phi(Y)]''$ or $S', S'', [d\Phi(Y)]'$ are $\mathbb{R}$-linearely independent. We may assume the first case without loss of generality. We can take $Q = d\Phi(W)$, $W \in \mathbb{C}^\infty(T^+M)$ satisfying $Q'' \neq 0$ and $G(Q'', S') = 0$. In fact, such a vector is $W := \alpha Z - Y$, with $\alpha = [G(V,U) - iG(V,JU)]/\|U\|^2$, where (G,J) denote the Kähler sructure on N and $U = \mathrm{Re}(S')$, $V = \mathrm{Re}([d\Phi(Y)]'') \in \mathbb{C}^\infty(TN)$. From (2.7), we obtain:

$$
\begin{aligned}
0 = R^N(S,Q,\overline{Q},\overline{S'}) &= R^N(S,Q,\overline{Q}',\overline{S'}) \\
&= R^N(S',Q'',\overline{Q''},\overline{S'}) - R^N(Q',S'',\overline{Q''},\overline{S'}) \\
&= k\{\|S'\|^2\|Q''\|^2 - G(Q',\overline{S}'')G(S'',\overline{Q}')\},
\end{aligned}
$$

where $k = -\frac{c}{2}$. By interchanging Z and $W \in \mathbb{C}^\infty(T^+M)$, we obtain:

$$0 = R^N(S,Q,\overline{S},\overline{Q'}) = k\{G(S',\overline{Q'})G(Q'',\overline{S}') - |G(S'',Q')|^2 - \|Q'\|^2\|\overline{S}'\|^2\}.$$

The last two relations imply:

$$\begin{aligned}0 &= R^N(S,Q,\overline{Q},\overline{S'}) - \overline{R^N(S,Q,\overline{S},\overline{Q'})}\\ &= k\{\|S'\|^2\|Q''\|^2 + |G(S'',Q')|^2 + \|Q'\|^2\|\overline{S}'\|^2\},\end{aligned}$$

from which we draw the false conclusion that either S' or $Q'' = 0$.

Step 2. We prove $d\Phi(T^0M) = 0$ or equivalently, $d\Phi(\mathrm{Ker}F) = 0$. We may assume (2.9) without loss of generality. Since we have $\mathrm{rank}(d\Phi\circ F) \geq 3$, there exists $S \neq 0$, $S = d\Phi(Z) \in \mathbb{C}^\infty(T^{1,0}N)$, $Z \in \mathbb{C}^\infty(T^+M)$. If $\xi \in \mathbb{C}^\infty(\mathrm{Ker}F)$, we put $Q = d\Phi(\xi) \in T^{\mathbb{C}}N$ and then $Q = \overline{Q}$. From (2.7) and (2.8), we obtain: $0 = R^N(S,Q,\overline{Q},\overline{S}) = R^N(S,Q'',Q',\overline{S}) = k\|S\|^2\|Q'\|^2$. Then $Q' = 0$ and hence $Q = 0$, which complete the proof. □

Example 2.6. Let M be a totally geodesic hypersurface of $\mathbb{CP}^{n+1}$ endowed with the f-structure given by Corollary 1.5. Then, the natural projection $\pi : M \longrightarrow \mathbb{CP}^n$ provides an example for Theorem 2.4.

Corollary 2.7. *Let* (M,g,F) *be a manifold with Riemannian integrable* f*-structure satisfying condition* $\tilde{A}$*. Then, for any* f*-pluriharmonic map* $\Phi : M \longrightarrow N(c)$ *into a real space form* ($c \neq 0$)*, the conditions* (2.6) *and* $\mathrm{rank}(d\Phi\circ F) \geq 3$ *can not occur simultaneously.*

Proof. In the same way as in [11], we let $\psi : N(c) \longrightarrow \tilde{N}(4c)$ be a totally real totally geodesic immersion into a complex space form. Then, $\psi\circ\Phi$ is a non $\pm f$-holomorphic map and the statement follows from Theorem 2.4. □

§3. Complex sectional curvature

In Theorem 2.4 (resp. Corollary 2.7) the notion of holomorphic sectional curvature (resp. sectional curvature) was involved. This section deals with the notion of complex sectional curvature which was introduced by T. Siu for the proof of his strong rigidity theorem [9].

Let (N,G) be a Riemannian manifold and let $y \in N$. A plan $\pi = \mathrm{span}_{\mathbb{C}}\{S,Q\} \subset T^{\mathbb{C}}_yN$ is nondegenerate (resp. degenerate) if $\dim_{\mathbb{C}}\pi = 2$ (resp. ≤ 1).

The *complex sectional curvature* associated to a nondegenerate plan π is defined by:

$$K^{\mathbb{C}}(\pi) = \frac{R^N(S,Q,\overline{S},\overline{Q})}{\|S\|^2\|Q\|^2 - |G(S,Q)|^2} \tag{3.1}$$

We will say that N is of *strictly negative complex sectional curvature* if $K^{\mathbb{C}}(\pi) < 0$ for any non degenerate plan $\pi \subset T^{\mathbb{C}}_yN$ and any $y \in N$. This notion corresponds to what Sampson calls in [8] strongly negative Hermitian curvature. An example of this notion is any manifold of constant negative sectional curvature, [7].

Theorem 3.1. *(Sampson's theorem) For any harmonic map $\Phi : M \longrightarrow N$, where M is compact Kähler and N is a compact manifold with strictly negative complex sectional curvature at every point, the rank $d_p\Phi$ at any point $p \in M$ is at most 2.*

Our aim is to obtain the conclusion, by replacing the compactness conditions on M and N.

Proposition 3.2. *Let (M, g, F) be a manifold with Riemannian integrable f-structure satisfying condition $\tilde{A}$ and let N be a Riemannian manifold with strictly negative (resp. strictly positive) complex sectional curvature at every point. If $\Phi : M \longrightarrow N$ is an f-pluriharmonic map satisfying (2.6), then* $\mathrm{rank}(d\Phi \circ F) \leq 2$.

Proof. If we suppose the contrary, then $\dim_{\mathbb{R}} d\Phi(T^+M) = \mathrm{rank}(d\Phi \circ F) \geq 3$ at a certain point $p \in M$. Therefore, there exist $Z, W \in (T^+M)_p$ such that the plan $\pi = \mathrm{span}_{\mathbb{C}}\{Z, W\} \subset T^{\mathbb{C}}_{\Phi(p)}N$ is nondegenerate. From Lemma 2.5 and (2.6), it follows $K^{\mathbb{C}}(\pi) = 0$, which contradicts the above hypothesis. □

Corollary 3.3. *If $\Phi : M \longrightarrow N$ is a pluriharmonic map from a Kähler manifold into a Riemannian manifold with strictly negative (resp. strictly positive) complex sectional curvature at every point, then* $\mathrm{rank}(d\Phi) \leq 2$.

References

[1] A. Bejancu, *Geometry of CR-manifolds*, Reidel Dordrecht, 1986.

[2] D. Blair, *Contact manifolds in Riemannian geometry*, Springer - Verlag, 1976.

[3] F. E. Burstall, *Non-linear functional analysis and harmonic maps*, thesis, University of Warwick, 1984.

[4] J. Eells, L. Lemaire, *Another report on harmonic maps*, Bull. London Math. Soc. 20 (1988), 385-524.

[5] Y. Ohnita, G. Vali, *Pluriharmonic maps into compact Lie groups and factorization into unitons*, Proc. London Math. Soc. 61 (1990), 546-570.

[6] J. K. Rawnsley, *f-structures, f-twistor spaces and harmonic maps*, Sem. Geom. L. Bianchi II, 1984, Lecture Notes in Math. 1164 (Springer Berlin, 1985), 85-159.

[7] J. H. Sampson, *Harmonic maps in Kähler geometry*, Harmonic maps and Minimal Immersions, CIME conf. 1984, Lecture Notes in Math. 1161 (Springer Berlin, 1985), 193-205.

[8] J. H. Sampson, *Applications of harmonic maps to Kähler geometry*, Contemp. Math. 49 (1986), 125-134.

[9] Y. T. Siu, *The complex analyticity of harmonic maps and the strong rigidity of compact Kähler manifolds*, Ann. of Math. 112 (1980), 73-111.

[10] K. Takegoshi, *A non-existence theorem for pluriharmonic maps of finite energy*, Math. Z, 192 (1986), 21-27.

[11] S. Udagawa, *Pluriharmonic maps and minimal immersions of Kähler manifolds*, J. London Math. Soc. 37 (1988), 375-384.

[12] K. Yano, *On a structure defined by a tensor of type (1,1) satisfying* $f^3 + f = 0$, Tensor N.S. 14 (1963), 99-109.

Cornelia Livia Bejan
Seminar Matematic, Universitatea "Al. I. Cuza"
Iasi 700506, Romania
E-mail: bejan@math.tuiasi.ro

Michèle Benyounes
Département de Mathématiques, Université de Brest
6 avenue le Gorgeu, 29200 Brest-France
E-mail: Michele.Benyounes@univ-brest.fr