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Correction to: Mixed Problem for Weakly Hyperbolic Equations of Second Order with Degenerate First Order Boundary Condition

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The paper with the above title contains falsities. The falsities occur in formulas (3.18) and (3.19). We must change the choice of z_1 and z_2 in pp. 76 because we can not use formulas (3.18) and (3.19). For the correction, we have been able to choose z_1 and z_2 as $z_1=1$ and $z_2=(\tilde{c}(t, y')-1)/(\tilde{c}(t, y')+1)$ where $\tilde{c}(t, y')$ is the same function as the one in (3.12). In the paper with above title, z_1 and z_2 were pseudo differential operators with respect to $y'=(y_2, \cdots, y_n)$ with parameters (t, σ). By the new choice of z_1 and z_2, we have a simple systematization which reduce the mixed problem (3.9) to the mixed problem for the symmetric hyperbolic pseudo differential system of first order with positive boundary condition. Also, we can easily obtain (3.42) and (3.57) for the corrected system by the same method as the one in the paper with the above title.

The new systematization for (3.9) is as follows:

By Lemma 3.3, we have

$$(1)\quad \begin{cases} \inf\limits_{(t,y')\in[0,T]\times R^{n-1}} \tilde{c}_1(t, y')>0 \\ \inf\limits_{(t,y',\eta')\in[0,T]\times R^{n-1}\times R^{n-1}} [\tilde{c}_1(t, y')^2-\{\tilde{b}_{11}(t, y', \eta')^2 \\ \qquad +(\tilde{c}_1(t, y')\tilde{b}_{12}(t, y', \eta')-\tilde{c}_2(t, y')\tilde{b}_{11}(t, y', \eta'))^2\}]>0 \end{cases}$$

where $\tilde{b}_{11}$, $\tilde{b}_{12}$, $\tilde{c}_1$ and $\tilde{c}_2$ are real valued functions, $\tilde{\alpha}_j(t, y')$, $\tilde{\beta}(t, y')$ and $d(\eta')$ are the same ones in (3.12), $\eta'=(\eta_2, \cdots, \eta_n)$ and

$$(2)\quad \begin{cases} \tilde{c}(t, y')=\tilde{\beta}(t, y')=\tilde{c}_1(t, y)+i\tilde{c}_2(t, y') \\ \sum\limits_{j=2}^{n} \tilde{\alpha}_j(t, y')\eta_j/d_1(\eta')=\tilde{b}_{11}(t, y', \eta')+i\tilde{b}_{12}(t, y', \eta') \\ d_1(\eta')=(d(\eta')^2+1)^{1/2}\ . \end{cases}$$

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Let Q_0 and Q_1 be

$$Q_0=\left(1+\frac{\tilde{h}_1(t,y)^2}{\tilde{a}_{11}(t,y)}\right)^{1/2}\left\{\frac{\partial}{\partial t}-\left(1+\frac{\tilde{h}_1(t,y)^2}{\tilde{a}_{11}(t,y)}\right)^{-1}\cdot(t+\delta)^k \times\left(\sum_{j=2}^{n}\tilde{h}_j(t,y)\frac{\partial}{\partial y_j}-\frac{\tilde{h}_1(t,y)}{\tilde{a}_{11}(t,y)}\sum_{j=2}^{n}\tilde{a}_{1j}(t,y)\frac{\partial}{\partial y_j}\right)\right\} \tag{3}$$

and

$$Q_1=\frac{1}{\sqrt{\tilde{a}_{11}(t,y)}}\left\{\tilde{a}_{11}(t,y)\frac{\partial}{\partial y_1}+\sum_{j=2}^{n}\tilde{a}_{1j}(t,y)\frac{\partial}{\partial y_j}+\frac{\tilde{h}_1(t,y)}{(t+\delta)^k}\frac{\partial}{\partial t}\right\}+\frac{\tilde{\gamma}(t,y')}{(t+\delta)^k} \tag{4}$$

respectively. Also, let Q_2 be a pseudo differential operator with respect to $y'=(y_2,\cdots,y_n)$ with the symbol

$$\sigma(Q_2)=i\left[\sum_{i,j=2}^{n}\tilde{a}_{ij}(t,y)\eta_i\eta_j-\frac{1}{\tilde{a}_{11}(t,y)}\left(\sum_{j=2}^{n}\tilde{a}_{1j}(t,y)\eta_j\right)^2 +\left(1+\frac{\tilde{h}_1(t,y)^2}{\tilde{a}_{11}(t,y)}\right)^{-1}\cdot\left(\sum_{j=2}^{n}\tilde{h}_j(t,y)\eta_j-\frac{\tilde{h}_1(t,y)}{\tilde{a}_{11}(t,y)}\sum_{j=2}^{n}\tilde{a}_{1j}(t,y)\eta_j\right)^2+1\right]^{1/2}. \tag{5}$$

We use the notations $v=\bar{\varphi}\cdot\bar{u}_\delta$ and $w=\psi\cdot\bar{u}_\delta$ where $\psi\in C_0^\infty(\boldsymbol{R}^n)$ and $\psi=1$ on $\operatorname{supp}[\bar{\varphi}]$. We set

$$U=\begin{pmatrix}U_1\\U_2\\U_3\\U_4\\U_5\\U_6\\U_7\end{pmatrix}=\begin{pmatrix}Q_0v-(t+\delta)^kQ_1v+(t+\delta)^k[pQ_2v-iqQ_2v]\\ Q_0v+(t+\delta)^kQ_1v+(t+\delta)^k[pQ_2v+iqQ_2v]\\ \sqrt{2}\,i(t+\delta)^krQ_2v\\ Q_0v-(t+\delta)^kQ_1v+(t+\delta)^kz[pQ_2v+iqQ_2v]\\ z[Q_0v+(t+\delta)^kQ_1v]+(t+\delta)^k[pQ_2v-iqQ_2v]\\ i\sqrt{1+|z|^2}(t+\delta)^krQ_2v\\ v\end{pmatrix} \tag{6}$$

where p, q and r are pseudo differential operators with respect to $y'=(y_2,\cdots,y_n)$ whose symbol are

$$\begin{cases}\sigma(p)=-\tilde{b}_{11}(t,y',\eta')/\tilde{c}_1(t,y')\\ \sigma(q)=(\tilde{c}_1(t,y')\tilde{b}_{12}(t,y',\eta')-\tilde{c}_2(t,y')\tilde{b}_{11}(t,y',\eta'))/\tilde{c}_1(t,y')\\ \sigma(r)=(1-\sigma(p)^2-\sigma(q)^2)^{1/2}\end{cases} \tag{7}$$

and $z=(\tilde{c}(t,y')-1)/(\tilde{c}(t,y')+1)$. By (1), we have

$$\begin{cases}\displaystyle\inf_{(t,y',\eta')\in[0,T]\times R^{n-1}\times R^{n-1}}\{1-\sigma(p)^2-\sigma(q)^2\}>0\\ \displaystyle\sup_{(t,y')\in[0,T]\times R^{n-1}}|z(t,y')|<1\,.\end{cases} \tag{8}$$

Then, the problem (3.9) is transformed into the problem:

$$
(9) \qquad \begin{cases} MU_t = AU_{y_1} + \sum_{j=2}^{n} B_j U_{y_j} + D_1 Q_2 U + D_2 Q_2 U + \dfrac{1}{t+\delta} EU + KV + F \\ U(0, y) = 0 \\ PU|_{y_1=0} = G \\ (t, y) \in (0, T) \times \boldsymbol{R}_+^n \end{cases}
$$

where

$$
M = \left(1 + \frac{\tilde{h}_1(t, y)^2}{\tilde{a}_{11}(t, y)}\right)^{1/2} I - \frac{\tilde{h}_1(t, y)}{\sqrt{\tilde{a}_{11}(t, y)}} \begin{pmatrix} -1 & & & & & & \\ & 1 & & & & & 0 \\ & & 0 & & & & \\ & & & -1 & & & \\ & & & & 1 & & \\ & 0 & & & & \dfrac{1-|z|^2}{1+|z|^2} & \\ & & & & & & 1 \end{pmatrix}
$$

$$
A = (t+\delta)^k \sqrt{\tilde{a}_{11}(t, y)} \begin{pmatrix} -1 & & & & & & \\ & 1 & & & & & 0 \\ & & 0 & & & & \\ & & & -1 & & & \\ & & & & 1 & & \\ & 0 & & & & \dfrac{1-|z|^2}{1+|z|^2} & \\ & & & & & & 1 \end{pmatrix}
$$

$$
B_j = \frac{\tilde{a}_{1j}(t, y)}{\tilde{a}_{11}(t, y)} A + \left(1 + \frac{\tilde{h}_1(t, y)^2}{\tilde{a}_{11}(t, y)}\right)^{-1/2} (t+\delta)^k \left(\tilde{h}_j(t, y) - \frac{\tilde{h}_1(t, y)}{\tilde{a}_{11}(t, y)} \tilde{a}_{1j}(t, y)\right) I
$$

$$(j = 2, \cdots, n)$$

$$
D_1 = (t+\delta)^k \left(\begin{array}{ccc|cccc} 0 & p & 0 & & & & \\ p & 0 & 0 & & & 0 & \\ 0 & 0 & -p & & & & \\ \hline & & & 0 & p & 0 & 0 \\ & & & p & 0 & 0 & 0 \\ & 0 & & 0 & 0 & -\dfrac{2p \operatorname{Re} z}{1+|z|^2} & 0 \\ & & & 0 & 0 & 0 & 0 \end{array}\right)
$$

$$D_2=(t+\delta)^k\left(\begin{array}{ccc|cccc} 0 & -iq & -\frac{ir}{\sqrt{2}} & & & & \\ iq & 0 & -\frac{ir}{\sqrt{2}} & & 0 & & \\ \frac{ir}{\sqrt{2}} & \frac{ir}{\sqrt{2}} & 0 & & & & \\ \hline & & & 0 & iq & -\frac{ir}{\sqrt{1+|z|^2}} & 0 \\ & 0 & & -iq & 0 & -\frac{izr}{\sqrt{1+|z|^2}} & 0 \\ & & & \frac{ir}{\sqrt{1+|z|^2}} & \frac{i\bar{z}r}{\sqrt{1+|z|^2}} & \frac{2q\cdot \mathrm{Im}\, z}{1+|z|^2} & 0 \\ & & & 0 & 0 & 0 & 0 \end{array}\right)$$

E ⋯ a 7×7 pseudo differential system which has the property that for $\sigma(E)=(e_{ij})$, the following conditions holds:

(i) $e_{ij}(t, y, \eta')\in C^\infty([0, T]\times \overline{R^n_+}\times R^{n-1})$,

(ii) for any $\theta=(\theta_1, \theta_2, \theta_3, \theta_4)$, there is a positive constant $C_\theta^{(i,j)}$ such that

$$\left|\left(\frac{\partial}{\partial t}\right)^{\theta_1}\left(\frac{\partial}{\partial y_1}\right)^{\theta_2}\left(\frac{\partial}{\partial y'}\right)^{\theta_3}\left(\frac{\partial}{\partial \eta'}\right)^{\theta_4} e_{ij}\right|\leqq C_\theta^{(i,j)}\langle\eta'\rangle^{-|\theta_4|}$$

where $\langle\eta'\rangle=(\sum_{j=2}^n \eta_j^2+1)^{1/2}$.

K ⋯ a $7\times(n+3)$ pseudo differential system which has the same property as E.

$$V={}^t(w, w_t, (t+\delta)^k w_{y_1}, \cdots, (t+\delta)^k w_{y_n}, (t+\delta)^k Q_2 w)$$

$$F={}^t(\bar{\varphi}\cdot\bar{f}_1, \bar{\varphi}\cdot\bar{f}_1, 0, \bar{\varphi}\cdot\bar{f}_1, z\cdot\bar{\varphi}\cdot\bar{f}_1, 0, 0)$$

$$P=\begin{pmatrix} 1 & z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$G={}^t\left(-\frac{2}{\tilde{c}(t, y)+1}(\bar{\varphi}\cdot\bar{g}_1+(t+\delta)^k T_0 w),\ -\frac{2}{\tilde{c}(t, y)+1}(\bar{\varphi}\cdot\bar{g}_1+(t+\delta)^k T_0 w)\right)$$

$$(T_0\in S^0(1))$$

and

(10) $\qquad ((AU, U))\geqq C(t+\delta)^k((U, U))$ for any $U\in \mathrm{Ker}\, P$.

Remark. See [1: pp. 194–197] for the above systematization. Also, the above systematization is useful to treat the mixed problem for regularly hyperbolic equations of second order with the uniform Lopatinski boundary condition.

Reference

[1] M. TANIGUCHI, Mixed problem for hyperbolic equations of second order in a domain with a corner, Tokyo J. Math., **5** (1982), 183–211.

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