Almost homoclinics for nonautonomous second order Hamiltonian systems by a variational approach *

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Abstract

In this paper we shall be concerned with the existence of almost homoclinic solutions of the Hamiltonian system $\ddot{q} + V_q(t, q) = f(t)$, where $t \in \mathbb{R}$, $q \in \mathbb{R}^n$ and $V(t, q) = -\frac{1}{2}(L(t)q, q) + W(t, q)$. It is assumed that L is a continuous matrix valued function such that $L(t)$ are symmetric and positive definite uniformly with respect to t. A map W is C^1-smooth, $W_q(t, q) = o(|q|)$, as $q \to 0$ uniformly with respect to t and $W(t, q)|q|^{-2} \to \infty$, as $|q| \to \infty$. Moreover, $f \neq 0$ is continuous and sufficiently small in $L^2(\mathbb{R}, \mathbb{R}^n)$. It is proved that this Hamiltonian system possesses a solution $q_0 \colon \mathbb{R} \to \mathbb{R}^n$ such that $q_0(t) \to 0$, as $t \to \pm\infty$. Since $q \equiv 0$ is not a solution of our system, q_0 is not homoclinic in a classical sense. We are to call such a solution almost homoclinic. It is obtained as a weak limit of a sequence of almost critical points of an appropriate action functional I.

1 Introduction

In this work we will look more closely at the second order Hamiltonian system:

$$\ddot{q} + V_q(t, q) = f(t), \tag{1}$$

where $t \in \mathbb{R}$, $q \in \mathbb{R}^n$ and functions $V \colon \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$, $f \colon \mathbb{R} \to \mathbb{R}^n$ satisfy:

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(V_1) $V(t,x) = -\frac{1}{2}(L(t)x, x) + W(t,x)$ for all $t \in \mathbb{R}$, $x \in \mathbb{R}^n$,

(V_2) L is a continuous matrix valued function such that $L(t)$ are symmetric and positive definite uniformly with respect to $t \in \mathbb{R}$, i.e. there is $\alpha > 0$ such that
$$(L(t)x, x) \geq \alpha |x|^2$$
for all $x \in \mathbb{R}^n$ and $t \in \mathbb{R}$,

(V_3) W is C^1-smooth and there exists $\mu > 2$ such that
$$0 < \mu W(t,x) \leq (W_q(t,x), x)$$
for all $x \in \mathbb{R}^n \setminus \{0\}$ and $t \in \mathbb{R}$,

(V_4) $W_q(t,x) = o(|x|)$, as $x \to 0$ uniformly with respect to t,

(V_5) there is a continuous map $\overline{W}\colon \mathbb{R}^n \to \mathbb{R}$ such that
$$W(t,x) \leq \overline{W}(x)$$
for all $t \in \mathbb{R}$, $x \in \mathbb{R}^n$,

(V_6) $f\colon \mathbb{R} \to \mathbb{R}^n$ is continuous and $f \neq 0$.

Here $(\cdot,\cdot)\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the standard scalar product in $\mathbb{R}^n$ and $|\cdot|$ is the induced norm.

Let us remark that (V_3)-(V_4) implies that
$$W(t,x) = o(|x|^2), \tag{2}$$
as $x \to 0$ uniformly with respect to t. Moreover, from (V_3) it follows that a mapping
$$(0,\infty) \ni s \longrightarrow W(t, s^{-1}x)s^{\mu}$$
is nonincreasing for all $t \in \mathbb{R}$ and $x \neq 0$. Hence for every $t \in \mathbb{R}$,
$$W(t,x) \leq W\left(t, \frac{x}{|x|}\right) |x|^{\mu}, \text{ if } 0 < |x| \leq 1 \tag{3}$$
and
$$W(t,x) \geq W\left(t, \frac{x}{|x|}\right) |x|^{\mu}, \text{ if } |x| \geq 1. \tag{4}$$
From (4) we conclude that W grows at a superquadratic rate, as $|x| \to \infty$. That is for each $t \in \mathbb{R}$,
$$\frac{W(t,x)}{|x|^2} \to \infty, \text{ as } |x| \to \infty.$$
By the assumptions (V_1)-(V_6), $q \equiv 0$ is not a solution of (1). Thus our Hamiltonian system does not possess a solution homoclinic to 0 in a classical meaning. However, we can still ask for the existence of solutions emanating from 0 and terminating at 0.

Definition 1.1. *We will say that a solution q of* (1) *is almost homoclinic* (*to* 0) *if* $q(t) \to 0$, *as* $t \to \pm\infty$.

Let us define

$$E := \{q \in W^{1,2}(\mathbb{R}, \mathbb{R}^n)\colon \int_{-\infty}^{\infty} \Big(|\dot{q}(t)|^2 + (L(t)q(t), q(t)) \Big)\, dt < \infty\}.$$

Then E is a Hilbert space under the norm

$$\|q\|_E^2 := \int_{-\infty}^{\infty} \Big(|\dot{q}(t)|^2 + (L(t)q(t), q(t)) \Big)\, dt.$$

Moreover, for $q \in E$,

$$\|q\|_{W^{1,2}(\mathbb{R},\mathbb{R}^n)} \leq \beta \|q\|_E, \tag{5}$$

where $\beta^{-1} := \sqrt{\min\{1, \alpha\}}$. Set

$$M := \max\{\overline{W}(x)\colon x \in \mathbb{R}^n \ \wedge \ |x| = 1\}.$$

Then $M > 0$, by (V_3) and (V_5). Suppose that

(V_7)

$$M < \frac{1}{2\beta^2} \quad \text{and} \quad \|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} < \frac{\sqrt{2}}{2} \left(\frac{1}{2\beta^2} - M \right).$$

Let us remark that if $\alpha \geq 1$ then $\beta = 1$ and, in consequence, $M < \frac{1}{2}$ and $\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} < \frac{\sqrt{2}}{2}\left(\frac{1}{2} - M\right)$. We will prove the following theorem.

Theorem 1.1. *If the conditions* (V_1)-(V_7) *are satisfied then the Hamiltonian system* (1) *has an almost homoclinic solution* $q_0 \in E$.

Many authors have studied the existence of homoclinic solutions of Hamiltonian systems. For a treatment of this subject we refer the reader for example to [1, 2, 3, 4, 5, 9, 11, 12, 13]. This work is motivated by [10] in which P. Rabinowitz and K. Tanaka received the following result.

Theorem 1.2 (see [10], Th. 5.4, p. 491)**.** *Suppose that* $V\colon \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ *satisfies* (V_1), (V_3)-(V_4) *and*

(V_8) $L \in C(\mathbb{R}, \mathbb{R}^{n^2})$ *is a function such that* $L(t)$ *is a positive definite symmetric matrix for all* $t \in \mathbb{R}$ *and the smallest eigenvalue of* $L(t) \to \infty$, *as* $|t| \to \infty$, *i.e.*

$$\inf_{|x|=1} (L(t)x, x) \to \infty, \text{ as } |t| \to \infty,$$

(V_9) *there is* $\overline{W} \in C(\mathbb{R}^n, \mathbb{R})$ *such that*

$$W(t, x) + |W_q(t, x)| \leq \overline{W}(x)$$

for all $x \in \mathbb{R}^n$, $t \in \mathbb{R}$.

Then the Hamiltonian system

$$\ddot{q} + V_q(t,q) = 0$$

has a nontrivial homoclinic to 0 *solution* $q \in E$.

Our theorem extends the result of Rabinowitz and Tanaka to the case where f is nonzero. We see at once that (V_9) implies (V_5) and it is easy to check that (V_8) gives (V_2). From (V_8) it follows that there exists $r > 0$ such that

$$|t| > r \Longrightarrow \inf_{|x|=1} (L(t)x,x) > 1.$$

Set

$$\gamma := \min_{|t|\le r} \inf_{|x|=1} (L(t)x,x).$$

Since $L(t)$ is positive definite for each $t \in \mathbb{R}$, we get $\gamma > 0$. For all $x \in \mathbb{R}^n$ and $t \in \mathbb{R}$ we have

$$(L(t)x,x) \ge \inf_{|y|=1} (L(t)y,y)|x|^2 \ge \min\{1,\gamma\}|x|^2,$$

which yields (V_2) with $\alpha = \min\{1,\gamma\}$.

Similarly to [10] our solution is obtained by variational methods. Namely, applying Ekeland's variational principle we receive a sequence $\{q_k\}_{k\in\mathbb{N}}$ weakly convergent in E such that its weak limit is an almost homoclinic solution of (1).

In [6, 7] we also studied almost homoclinic solutions of Hamiltonian systems. There we considered the case where V is periodic with respect to $t \in \mathbb{R}$.

2 Proof of Theorem 1.1

At first, for the convenience of the reader we recall some inequalities which hold for all $q \in E$, thus making our exposition self-contained. We start with a result which the proof can be found for example in [6].

Fact 2.1 (see [6], Fact 2.8, p. 385). *Let* $q\colon \mathbb{R} \to \mathbb{R}^n$ *be a continuous mapping such that* $\dot{q} \in L^2_{loc}(\mathbb{R},\mathbb{R}^n)$. *For every* $t \in \mathbb{R}$, *the following inequality holds:*

$$|q(t)| \le \sqrt{2}\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left(|q(s)|^2 + |\dot{q}(s)|^2\right) ds\right)^{\frac{1}{2}}. \tag{6}$$

The estimation (6) implies that for each $q \in W^{1,2}(\mathbb{R},\mathbb{R}^n)$,

$$\|q\|_{L^\infty(\mathbb{R},\mathbb{R}^n)} \le \sqrt{2}\|q\|_{W^{1,2}(\mathbb{R},\mathbb{R}^n)}. \tag{7}$$

Combining (7) with (5), we get

$$\|q\|_{L^\infty(\mathbb{R},\mathbb{R}^n)} \le \sqrt{2}\beta\|q\|_E \tag{8}$$

for each $q \in E$. By (7), if $p \geq 2$, then for each $q \in W^{1,2}(\mathbb{R}, \mathbb{R}^n)$,

$$\begin{aligned} \int_{-\infty}^{\infty} |q(t)|^p dt &\leq \|q\|^{p-2}_{L^\infty(\mathbb{R},\mathbb{R}^n)} \int_{-\infty}^{\infty} |q(t)|^2 dt \\ &\leq 2^{\frac{p-2}{2}} \|q\|^{p-2}_{W^{1,2}(\mathbb{R},\mathbb{R}^n)} \int_{-\infty}^{\infty} |q(t)|^2 dt \\ &\leq 2^{\frac{p-2}{2}} \|q\|^{p}_{W^{1,2}(\mathbb{R},\mathbb{R}^n)}. \end{aligned}$$

Hence, if $p \geq 2$,

$$\|q\|_{L^p(\mathbb{R},\mathbb{R}^n)} \leq 2^{\frac{p-2}{2p}} \|q\|_{W^{1,2}(\mathbb{R},\mathbb{R}^n)} \tag{9}$$

for each $q \in W^{1,2}(\mathbb{R}, \mathbb{R}^n)$ and, in addition, if $\|q\|_{L^\infty(\mathbb{R},\mathbb{R}^n)} \leq 1$, then

$$\|q\|^p_{L^p(\mathbb{R},\mathbb{R}^n)} \leq \|q\|^2_{L^2(\mathbb{R},\mathbb{R}^n)}. \tag{10}$$

For $q \in E$, let

$$I(q) := \frac{1}{2}\|q\|^2_E - \int_{-\infty}^{\infty} W(t, q(t))dt + \int_{-\infty}^{\infty} (f(t), q(t))dt.$$

Then $I \in C^1(E, \mathbb{R})$ and it is easy to verify that any critical point of I on E is a classical solution of (1). Moreover,

$$\begin{aligned} I'(q)w &= \int_{-\infty}^{\infty} ((\dot{q}(t), \dot{w}(t)) + (L(t)q(t), w(t)))\, dt \\ &\quad - \int_{-\infty}^{\infty} (W_q(t, q(t)), w(t))dt + \int_{-\infty}^{\infty} (f(t), w(t))dt \end{aligned}$$

for all $q, w \in E$.

We will prove that I has a critical point by the use of Ekeland's variational principle. Therefore, we state this theorem precisely.

Theorem 2.2 (see [8], Th. 4.3, p. 77)**.** *Let K be a compact metric space, $K_0 \subset K$ a closed subset, X a Banach space, $\chi \in C(K_0, X)$ and let us define the complete metric space $\mathcal{M}$ by*

$$\mathcal{M} := \{g \in C(K, X) \colon g(s) = \chi(s) \text{ if } s \in K_0\}$$

with the usual distance. Let $\varphi \in C^1(X, \mathbb{R})$ and let us define

$$c := \inf_{g \in \mathcal{M}} \max_{s \in K} \varphi(g(s))$$

and

$$c_1 := \max_{\chi(K_0)} \varphi.$$

If $c > c_1$, then for each $\varepsilon > 0$ and for each $h \in \mathcal{M}$ such that

$$\max_{s \in K} \varphi(h(s)) \leq c + \varepsilon,$$

there exists $v \in X$ such that

$$c - \varepsilon \leq \varphi(v) \leq \max_{s \in K} \varphi(h(s)),$$

$$dist(v, h(K)) \leq \varepsilon^{\frac{1}{2}},$$
$$\|\varphi'(v)\|_{X^*} \leq \varepsilon^{\frac{1}{2}}.$$

The proof of Theorem 1.1 will be divided into a sequence of lemmas.

Lemma 2.3. *There are $\varrho > 0$ and $\lambda > 0$ such that if $\|q\|_E = \varrho$, then $I(q) \geq \lambda$.*

Proof. For all $q \in E$,

$$I(q) \geq \frac{1}{2}\|q\|_E^2 - \int_{-\infty}^{\infty} W(t, q(t))dt - \beta\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)}\|q\|_E. \tag{11}$$

Set

$$\varrho := \frac{\sqrt{2}}{2\beta}. \tag{12}$$

Assume that $0 < \|q\|_E \leq \varrho$. Then (8) implies that $0 < \|q\|_{L^\infty(\mathbb{R},\mathbb{R}^n)} \leq 1$. Applying (V_5), (3) and (10), we get

$$\begin{aligned}\int_{-\infty}^{\infty} W(t, q(t))dt &\leq \int_{-\infty}^{\infty} W\left(t, \frac{q(t)}{|q(t)|}\right)|q(t)|^\mu dt \\ &\leq \int_{-\infty}^{\infty} \overline{W}\left(\frac{q(t)}{|q(t)|}\right)|q(t)|^\mu dt \\ &\leq M\int_{-\infty}^{\infty} |q(t)|^\mu dt = M\|q\|^\mu_{L^\mu(\mathbb{R},\mathbb{R}^n)} \\ &\leq M\|q\|^2_{L^2(\mathbb{R},\mathbb{R}^n)} \leq M\beta^2\|q\|_E^2.\end{aligned}$$

Consequently, if $\|q\|_E \leq \varrho$, then

$$I(q) \geq \frac{1}{2}\|q\|_E^2 - M\beta^2\|q\|_E^2 - \beta\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)}\|q\|_E.$$

Thus for $\|q\|_E = \varrho$,

$$\begin{aligned}I(q) &\geq \left(\frac{1}{2} - M\beta^2\right)\varrho^2 - \beta\varrho\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} \\ &= \frac{1}{2}\left(\frac{1}{2\beta^2} - M\right) - \frac{\sqrt{2}}{2}\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} \equiv \lambda.\end{aligned}$$

From (V_7) we get that $\lambda > 0$, which completes the proof. ∎

Lemma 2.4. *Let ϱ be a constant defined by* (12)*. Then there exists $Q \in E$ such that $\|Q\|_E > \varrho$ and $I(Q) < 0$.*

Proof. Take $u \in C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ such that $|u(t)| = 1$ if $|t| \leq 1$ and $u(t) = 0$ if $|t| > 2$. Let us define m as follows:

$$m := \inf\{W(t, x) \colon |t| \leq 1 \,\wedge\, |x| = 1\}.$$

(V_3) implies that $m > 0$. By the use of (4), for every $\xi \geq 1$, we receive

$$\begin{aligned}\int_{-\infty}^{\infty} W(t,\xi u(t))dt &\geq \int_{-1}^{1} W(t,\xi u(t))dt \geq \int_{-1}^{1} W\left(t, \frac{u(t)}{|u(t)|}\right) |\xi u(t)|^{\mu} dt \\ &\geq m\xi^{\mu} \int_{-1}^{1} |u(t)|^{\mu} dt = 2m\xi^{\mu}.\end{aligned}$$

In consequence,

$$\begin{aligned}I(\xi u) &= \frac{1}{2}\xi^2 \|u\|_E^2 - \int_{-\infty}^{\infty} W(t,\xi u(t))dt + \xi \int_{-\infty}^{\infty} (f(t),u(t))dt \\ &\leq \frac{1}{2}\xi^2 \|u\|_E^2 - 2m\xi^{\mu} + \xi\beta \|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} \|u\|_E,\end{aligned}$$

and hence $I(\xi u) \to -\infty$, as $\xi \to \infty$. Thus if ξ is large enough, then $Q = \xi u$ satisfies the desired claim. ∎

From now on, let us define the complete metric space $\mathcal{M}$ by

$$\mathcal{M} := \{g \in C([0,1],E) \colon g(0) = 0 \ \wedge \ g(1) = Q\}$$

with the usual distance

$$d(g,h) := \max_{s \in [0,1]} \|g(s) - h(s)\|_E.$$

Let

$$c := \inf_{g \in \mathcal{M}} \max_{s \in [0,1]} I(g(s))$$

and

$$c_1 := \max\{I(0), I(Q)\},$$

where Q is determined by Lemma 2.4. We check at once that $c_1 = 0$. Moreover, combining Lemma 2.3 with Lemma 2.4 we have that $c \geq \lambda > 0$. Next, applying Theorem 2.2 we conclude that there exists a sequence $\{q_k\}_{k \in \mathbb{N}}$ in E such that

$$I(q_k) \to c \quad \wedge \quad I'(q_k) \to 0, \tag{13}$$

as $k \to \infty$. $\{q_k\}_{k \in \mathbb{N}}$ is so-called a sequence of almost critical points (compare [8], § 4.1, p. 75-80).

Lemma 2.5. *The sequence $\{q_k\}_{k \in \mathbb{N}}$ given by* (13) *possesses a weakly convergent subsequence in E.*

Proof. Since E is a Hilbert space, it is sufficient to show that $\{q_k\}_{k \in \mathbb{N}}$ is bounded. For all $k \in \mathbb{N}$, we have

$$\begin{aligned}I(q_k) - \frac{1}{\mu} I'(q_k) q_k &= \left(\frac{1}{2} - \frac{1}{\mu}\right) \|q_k\|_E^2 - \int_{-\infty}^{\infty} W(t,q_k(t))dt \\ &\quad + \frac{1}{\mu} \int_{-\infty}^{\infty} (W_q(t,q_k(t)), q_k(t))dt \\ &\quad + \left(1 - \frac{1}{\mu}\right) \int_{-\infty}^{\infty} (f(t), q_k(t))dt \\ &\geq \left(\frac{1}{2} - \frac{1}{\mu}\right) \|q_k\|_E^2 - \beta \left(1 - \frac{1}{\mu}\right) \|f\|_{L^2(\mathbb{R},\mathbb{R}^n)} \|q_k\|_E,\end{aligned}$$

by (V_3). From (13) we obtain that there is $k_0 \in \mathbb{N}$ such that for all $k > k_0$,

$$|I(q_k) - c| < 1 \quad \wedge \quad \|I'(q_k)\|_{E^*} < \mu.$$

Since $|I'(q_k)q_k| \leq \|I'(q_k)\|_{E^*}\|q_k\|_E$, we receive

$$I(q_k) - \frac{1}{\mu}I'(q_k)q_k \leq c + 1 + \|q_k\|_E$$

for all $k > k_0$. Consequently, we get

$$c + 1 + \|q_k\|_E \geq \left(\frac{1}{2} - \frac{1}{\mu}\right)\|q_k\|_E^2 - \beta\left(1 - \frac{1}{\mu}\right)\|f\|_{L^2(\mathbb{R},\mathbb{R}^n)}\|q_k\|_E \tag{14}$$

for all $k > k_0$. Since $\mu > 2$, the inequality (14) implies that $\{q_k\}_{k\in\mathbb{N}}$ is a bounded sequence in E. ■

Let $q_0 \in E$ be a weak limit of a weakly convergent subsequence of the sequence $\{q_k\}_{k\in\mathbb{N}}$. Without loss of generality we can assume that

$$q_k \rightharpoonup q_0 \quad \text{in} \quad E, \quad \text{as} \quad k \to \infty. \tag{15}$$

Lemma 2.6. *$q_0 \colon \mathbb{R} \to \mathbb{R}^n$ given by* (15) *is a desired almost homoclinic solution of the Hamiltonian system* (1)*.*

Proof. We have to show that $I'(q_0) \equiv 0$ and $q_0(t) \to 0$, as $t \to \pm\infty$.

Fix $u \in C_0^\infty(\mathbb{R},\mathbb{R}^n)$. There is $a > 0$ such that $\mathrm{supp}(u) \subset [-a,a]$. From (15) it follows that $q_k \to q_0$ uniformly on $[-a,a]$ and

$$\int_{-a}^{a}(\dot{q}_k(t),\dot{u}(t))dt \to \int_{-a}^{a}(\dot{q}_0(t),\dot{u}(t))dt,$$

as $k \to \infty$. Hence

$$\begin{aligned} I'(q_k)u &= \int_{-a}^{a}(\dot{q}_k(t),\dot{u}(t))dt + \int_{-a}^{a}(L(t)q_k(t),u(t))dt \\ &\quad - \int_{-a}^{a}(W_q(t,q_k(t)),u(t))dt + \int_{-a}^{a}(f(t),u(t))dt \xrightarrow{k\to\infty} \\ &\quad \int_{-a}^{a}(\dot{q}_0(t),\dot{u}(t))dt + \int_{-a}^{a}(L(t)q_0(t),u(t))dt \\ &\quad - \int_{-a}^{a}(W_q(t,q_0(t)),u(t))dt + \int_{-a}^{a}(f(t),u(t))dt = I'(q_0)u. \end{aligned}$$

On the other hand, by (13) we have $I'(q_k)u \to 0$, as $k \to \infty$. In consequence, we receive $I'(q_0)u = 0$. Since $C_0^\infty(\mathbb{R},\mathbb{R}^n)$ is dense in E, we have $I'(q_0) \equiv 0$. From (6) we conclude that $q_0(t) \to 0$, as $t \to \pm\infty$, which completes the proof. ■

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