HERMITE-HADAMARD TYPE INEQUALITIES FOR THE PRODUCT OF (α, m)-CONVEX FUNCTIONS

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Abstract. In the paper, the authors establish some Hermite-Hadamard type inequalities for the product of two (α, m)-convex functions.

1. Introduction

The following definitions are well-known in the literature.

Definition 1.1. *A function $f : I \subseteq \mathbb{R} = (-\infty, \infty) \to \mathbb{R}$ is said to be convex if*

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y) \tag{1.1}$$

holds for all $x, y \in I$ and $\lambda \in [0,1]$.

Definition 1.2 ([7])**.** *For $f : [0,b] \to \mathbb{R}$ and $m \in (0,1]$, if*

$$f(tx + m(1-t)y) \le t f(x) + m(1-t) f(y) \tag{1.2}$$

is valid for all $x, y \in [0,b]$ and $t \in [0,1]$, then we say that $f(x)$ is m-convex on $[0,b]$.

Definition 1.3 ([4])**.** *For $f : [0,b] \to \mathbb{R}$ and $(\alpha, m) \in (0,1] \times (0,1]$, if*

$$f(tx + m(1-t)y) \le t^\alpha f(x) + m(1-t^\alpha) f(y) \tag{1.3}$$

is valid for all $x, y \in [0,b]$ and $t \in [0,1]$, then we say that $f(x)$ is (α, m)-convex on $[0,b]$.

In recent decades, many inequalities of Hermite-Hadamard type for various kinds of convex functions have been established. Some of them may be recited as follows.

Theorem 1.4 ([3])**.** *Let $f : [a,b] \subseteq \mathbb{R}_0 = [0, \infty) \to \mathbb{R}$ be m-convex for fixed $m \in (0,1]$. Then*

$$\frac{1}{b-a} \int_a^b f(x)\,dx \le \min\left\{\frac{f(a) + m f(b/m)}{2}, \frac{m f(a/m) + f(b)}{2}\right\}. \tag{1.4}$$

Theorem 1.5 ([5])**.** *Let $f, g : [a, b] \subseteq \mathbb{R} \to \mathbb{R}_0$ be convex functions. Then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \frac{1}{3}M(a,b) + \frac{1}{6}N(a,b), \tag{1.5}$$

where $M(a,b) = f(a)g(a) + f(b)g(b)$ and $N(a,b) = f(a)g(b) + f(b)g(a)$.

Theorem 1.6 ([2])**.** *Let $f, g : \mathbb{R}_0 \to \mathbb{R}_0$ satisfy $fg \in L([a,b])$, where $0 \le a < b < \infty$. If f is m_1-convex and g is m_2-convex on $[a,b]$ for some fixed $m_1, m_2 \in (0,1]$, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \min\{M_1, M_2\}, \tag{1.6}$$

where

$$\begin{aligned} M_1 = & \frac{1}{3}\left[f(a)g(a) + m_1m_2 f\left(\frac{b}{m_1}\right)g\left(\frac{b}{m_2}\right)\right] \\ & + \frac{1}{6}\left[m_2 f(a) g\left(\frac{b}{m_2}\right) + m_1 f\left(\frac{b}{m_1}\right)g(a)\right] \end{aligned}$$

and

$$\begin{aligned} M_2 = & \frac{1}{3}\left[f(b)g(b) + m_1m_2 f\left(\frac{a}{m_1}\right)g\left(\frac{a}{m_2}\right)\right] \\ & + \frac{1}{6}\left[m_1 f\left(\frac{a}{m_1}\right)g(b) + m_2 f(b) g\left(\frac{a}{m_2}\right)\right]. \end{aligned}$$

Theorem 1.7 ([2])**.** *Let $f, g : \mathbb{R}_0 \to \mathbb{R}_0$ satisfy $fg \in L([a,b])$ with $0 \le a < b < \infty$. If f is (α_1, m_1)-convex and g is (α_2, m_2)-convex on $[a,b]$ for $(\alpha_1, m_1), (\alpha_2, m_2) \in (0,1] \times (0,1]$, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \min\{N_1, N_2\}, \tag{1.7}$$

where

$$\begin{aligned} N_1 = & \frac{f(a)g(a)}{\alpha_1+\alpha_2+1} + m_2\left[\frac{1}{\alpha_1+1} - \frac{1}{\alpha_1+\alpha_2+1}\right] f(a) g\left(\frac{b}{m_2}\right) \\ & - m_1\left(\frac{1}{\alpha_1+\alpha_2+1} - \frac{1}{\alpha_2+1}\right) g(a) f\left(\frac{b}{m_1}\right) \\ & + m_1m_2\left(1 - \frac{1}{\alpha_1+1} - \frac{1}{\alpha_2+1} + \frac{1}{\alpha_1+\alpha_2+1}\right) f\left(\frac{b}{m_1}\right) g\left(\frac{b}{m_2}\right) \end{aligned}$$

and

$$
\begin{aligned}
N_2 =& \frac{f(b)g(b)}{\alpha_1+\alpha_2+1} + m_2\left[\frac{1}{\alpha_1+1} - \frac{1}{\alpha_1+\alpha_2+1}\right] f(b) g\left(\frac{a}{m_2}\right) \\
& - m_1\left(\frac{1}{\alpha_1+\alpha_2+1} - \frac{1}{\alpha_2+1}\right) g(b) f\left(\frac{a}{m_1}\right) \\
& + m_1 m_2 \left(1 - \frac{1}{\alpha_1+1} - \frac{1}{\alpha_2+1} + \frac{1}{\alpha_1+\alpha_2+1}\right) f\left(\frac{a}{m_1}\right) g\left(\frac{a}{m_2}\right).
\end{aligned}
$$

In recent years, some inequalities of Hermite-Hadamard type for other kinds of convex functions were created in, [1, 6, 8, 9, 10, 11, 12] and closely related references therein.

The aim of this paper is to present some new inequalities of Hermite-Hadamard type for the product of two (α, m)-convex functions, which generalizes those results mentioned above.

2. Main Results

We are now in a position to establish some new integral inequalities of Hermite-Hadamard type for the product of two (α, m)-convex functions.

Theorem 2.1. *Let $f, g : \mathbb{R}_0 \to \mathbb{R}_0$ satisfy $f, fg^q \in L([a,b])$, where $0 \le a < b < \infty$ and $q \ge 1$. If f is (α_1, m_1)-convex on $\left[0, \frac{b}{m_1}\right]$ and g^q is (α_2, m_2)-convex on $\left[0, \frac{b}{m_2}\right]$ for $(\alpha_1, m_1), (\alpha_2, m_2) \in (0,1] \times (0,1]$, then*

$$
\begin{aligned}
& \frac{1}{b-a}\int_a^b f(x)g(x)\,dx \\
& \le \frac{[N(a,b;f,\alpha_1,m_1)]^{1-1/q} \min\left\{[M(a,b;f,g^q)]^{1/q}, [M(b,a;f,g^q)]^{1/q}\right\}}{(\alpha_1+1)[(\alpha_2+1)(\alpha_1+\alpha_2+1)]^{1/q}}.
\end{aligned}
$$

where

$$
N(a,b;f,\alpha,m) = f(a) + \alpha m f\left(\frac{b}{m}\right) \tag{2.1}
$$

and

$$
\begin{aligned}
M(a,b;f,g) =& (\alpha_1+1)(\alpha_2+1) f(a) g(a) + \alpha_2 m_2 (\alpha_2+1) f(a) g\left(\frac{b}{m_2}\right) \\
+& \alpha_1 m_1 (\alpha_1+1) g(a) f\left(\frac{b}{m_1}\right) + \alpha_1\alpha_2(\alpha_1+\alpha_2+2) m_1 m_2 f\left(\frac{b}{m_1}\right) g\left(\frac{b}{m_2}\right).
\end{aligned} \tag{2.2}
$$

Proof. Letting $x = ta + (1-t)b$ for $t \in [0,1]$ and making use of Hölder's integral inequality yields

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx = \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,dt$$
$$\le \left[\int_0^1 f(ta+(1-t)b)\,dt\right]^{1-1/q}\left[\int_0^1 f(ta+(1-t)b)g^q(ta+(1-t)b)\,dt\right]^{1/q}.$$

Further employing the conditions that f is (α_1, m_1)-convex on $\left[0, \frac{b}{m_1}\right]$ and g^q is (α_2, m_2)-convex on $\left[0, \frac{b}{m_2}\right]$ leads to

$$\int_0^1 f(ta+(1-t)b)\,dt \le \int_0^1 \left[t^{\alpha_1} f(a) + m_1(1-t^{\alpha_1}) f\left(\frac{b}{m_1}\right)\right] dt$$
$$= \frac{1}{\alpha_1+1} N(a,b;f,\alpha_1,m_1)$$

and

$$\int_0^1 f(ta+(1-t)b)g^q(ta+(1-t)b)\,dt$$
$$\le \int_0^1 \left[t^{\alpha_1} f(a) + m_1(1-t^{\alpha_1}) f\left(\frac{b}{m_1}\right)\right]\left[t^{\alpha_2} g^q(a) + m_2(1-t^{\alpha_2}) g^q\left(\frac{b}{m_2}\right)\right] dt$$
$$= \frac{1}{\alpha_1+\alpha_2+1} f(a)g^q(a) + \frac{\alpha_2 m_2}{(\alpha_1+1)(\alpha_1+\alpha_2+1)} f(a) g^q\left(\frac{b}{m_2}\right)$$
$$+ \frac{\alpha_1 m_1}{(\alpha_2+1)(\alpha_1+\alpha_2+1)} f\left(\frac{b}{m_1}\right) g^q(a)$$
$$+ \frac{\alpha_1\alpha_2(\alpha_1+\alpha_2+2)m_1 m_2}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)} f\left(\frac{b}{m_1}\right) g^q\left(\frac{b}{m_2}\right)$$
$$= \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)} M(a,b;f,g^q).$$

The proof of Theorem 2.1 is complete. □

Remark 2.2. *Theorem 2.1 applied to $q = 1$ becomes the inequality* (1.7).

Corollary 2.3. *Under conditions of Theorem 2.1,*

(1) *if $\alpha_1 = \alpha_2 = \alpha$, we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx$$
$$\le \frac{[N(a,b;f,\alpha,m_1)]^{1-1/q} \min\left\{[M(a,b;f,g^q)]^{1/q}, [M(b,a;f,g^q)]^{1/q}\right\}}{(\alpha+1)^{1+1/q}(2\alpha+1)^{1/q}};$$

(2) *if* $m_1 = m_2 = m$, *we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \frac{[N(a,b;f,\alpha_1,m)]^{1-1/q}\min\big\{[M(a,b;f,g^q)]^{1/q},[M(b,a;f,g^q)]^{1/q}\big\}}{(\alpha_1+1)[(\alpha_2+1)(\alpha_1+\alpha_2+1)]^{1/q}};$$

(3) *if* $\alpha_1 = \alpha_2 = m_1 = m_2 = 1$, *we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \frac{1}{2}\left(\frac{1}{3}\right)^{1/q}[f(a)+f(b)]^{1-1/q} \times \big[2f(a)g^q(a)+f(a)g^q(b)+f(b)g^q(a)+2f(b)g^q(b)\big]^{1/q}.$$

Theorem 2.4. *Let* $f, g : \mathbb{R}_0 \to \mathbb{R}_0$ *be such that* $f^q, g^{q/(q-1)} \in L([a,b])$, *where* $0 \le a < b < \infty$ *and* $q > 1$. *If* f^q *is* (α_1, m_1)*-convex on* $\big[0, \frac{b}{m_1}\big]$ *and* $g^{q/(q-1)}$ *is* (α_2, m_2)*-convex on* $\big[0, \frac{b}{m_2}\big]$ *for* $(\alpha_1, m_1), (\alpha_2, m_2) \in (0,1]\times(0,1]$, *then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \left[\frac{\min\{N(a,b;f^q,\alpha_1,m_1),N(b,a;f^q,\alpha_1,m_1)\}}{\alpha_1+1}\right]^{1/q} \times \left[\frac{\min\{N(a,b;g^{q/(q-1)},\alpha_2,m_2),N(b,a;g^{q/(q-1)},\alpha_2,m_2)\}}{\alpha_2+1}\right]^{1-1/q}, \quad (2.3)$$

where $N(a,b;f,\alpha,m)$ *is defined by* (2.1).

Proof. Taking $x = ta + (1-t)b$ for $t \in [0,1]$ and using Hölder's integral inequality generates

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx = \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,dt \le \left[\int_0^1 f^q(ta+(1-t)b)\,dt\right]^{1/q}\left[\int_0^1 g^{q/(q-1)}(ta+(1-t)b)\,dt\right]^{1-1/q}.$$

Utilizing properties that f^q is (α_1, m_1)-convex on $\big[0, \frac{b}{m_1}\big]$ and that $g^{q/(q-1)}$ is (α_2, m_2)-convex on $\big[0, \frac{b}{m_2}\big]$ discovers

$$\int_0^1 f^q(ta+(1-t)b)\,dt \le \int_0^1 \left[t^{\alpha_1}f^q(a)+m_1(1-t^{\alpha_1})f^q\left(\frac{b}{m_1}\right)\right]dt = \frac{1}{\alpha_1+1}N(a,b;f^q,\alpha_1,m_1).$$

Considering the symmetry of the estimated definite integral with respect to a and b results in

$$\int_0^1 f^q(ta+(1-t)b)\,dt \le \frac{\min\{N(a,b;f^q,\alpha_1,m_1),N(b,a;f^q,\alpha_1,m_1)\}}{\alpha_1+1}.$$

Similarly, we have

$$\int_0^1 g^{q/(q-1)}(ta+(1-t)b)\,dt \\ \le \frac{\min\bigl\{N(a,b;g^{q/(q-1)},\alpha_2,m_2),N(b,a;g^{q/(q-1)},\alpha_2,m_2\bigr\}}{\alpha_2+1}.$$

Theorem 2.4 is thus proved. □

Corollary 2.5. *Under conditions of Theorem 2.4, if* $\alpha_1=\alpha_2=m_1=m_2=1$*, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \frac{[f^q(a)+f^q(b)]^{1/q}\bigl[g^{q/(q-1)}(a)+g^{q/(q-1)}(b)\bigr]^{1-1/q}}{2}. \tag{2.4}$$

Theorem 2.6. *Let* $f,g:\mathbb{R}_0\to\mathbb{R}_0$ *be such that* $f^p g^{q-\ell(q-1)}, f^{(q-p)/(q-1)}g^\ell \in L([a,b])$*, where* $0\le a<b<\infty$*,* $q>1$*,* $q>p>0$*, and* $\frac{q}{q-1}>\ell>0$*. If* f^p *and* $f^{(q-p)/(q-1)}$ *are* (α_1,m_1)*-convex on* $\bigl[0,\frac{b}{m_1}\bigr]$ *and if* g^ℓ *and* $g^{q-\ell/(q-1)}$ *are* (α_2,m_2)*-convex on* $\bigl[0,\frac{b}{m_2}\bigr]$ *for* $(\alpha_1,m_1),(\alpha_2,m_2)\in(0,1]\times(0,1]$*, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx \le \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)} \\ \times\bigl[\min\bigl\{M\bigl(a,b;f^p,g^{q-\ell(q-1)}\bigr),M\bigl(b,a;f^p,g^{q-\ell(q-1)}\bigr)\bigr\}\bigr]^{1/q} \\ \times\bigl[\min\bigl\{M\bigl(a,b;f^{(q-p)/(q-1)},g^\ell\bigr),M\bigl(b,a;f^{(q-p)/(q-1)},g^\ell\bigr)\bigr\}\bigr]^{1-1/q},$$

where $M(a,b;f,g)$ *is defined by* (2.2)*.*

Proof. Letting $x=ta+(1-t)b$ for $t\in[0,1]$ and using Hölder's integral inequality, we obtain

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,dx = \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,dt \\ \le \left[\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,dt\right]^{1/q} \\ \times\left[\int_0^1 f^{(q-p)/(q-1)}(ta+(1-t)b)g^\ell(ta+(1-t)b)\,dt\right]^{1-1/q}.$$

Further by virtue of properties that the function f^p is (α_1, m_1)-convex on $\left[0, \frac{b}{m_1}\right]$ and that the function $g^{q-\ell/(q-1)}$ is (α_2, m_2)-convex on $\left[0, \frac{b}{m_2}\right]$, we have

$$\begin{aligned}
&\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,dt\\
&\quad\le \int_0^1 \left[t^{\alpha_1} f^p(a) + m_1(1-t^{\alpha_1}) f^p\left(\frac{b}{m_1}\right)\right]\\
&\qquad\times\left[t^{\alpha_2} g^{q-\ell(q-1)}(a) + m_2(1-t^{\alpha_2}) g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\right]dt\\
&\quad= \frac{1}{\alpha_1+\alpha_2+1} f^p(a) g^{q-\ell(q-1)}(a)\\
&\qquad+ \frac{\alpha_2 m_2}{(\alpha_1+1)(\alpha_1+\alpha_2+1)} f^p(a) g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\\
&\qquad+ \frac{\alpha_1 m_1}{(\alpha_2+1)(\alpha_1+\alpha_2+1)} f^p\left(\frac{b}{m_1}\right) g^{q-\ell(q-1)}(a)\\
&\qquad+ \frac{\alpha_1\alpha_2(\alpha_1+\alpha_2+2) m_1 m_2}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)} f^p\left(\frac{b}{m_1}\right) g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\\
&\quad= \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)} M\big(a,b; f^p, g^{q-\ell(q-1)}\big).
\end{aligned}$$

Changing the order of a and b in the above arguments reveals

$$\begin{aligned}
&\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,dt\\
&\qquad\le \frac{\min\big\{M\big(a,b; f^p, g^{q-\ell(q-1)}\big), M\big(b,a; f^p, g^{q-\ell(q-1)}\big)\big\}}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}
\end{aligned}$$

and

$$\begin{aligned}
&\int_0^1 f^{(q-p)/(q-1)}(ta+(1-t)b)g^{\ell}(ta+(1-t)b)\,dt\\
&\qquad\le \frac{\min\big\{M\big(a,b; f^{(q-p)/(q-1)}, g^{\ell}\big), M\big(b,a; f^{(q-p)/(q-1)}, g^{\ell}\big)\big\}}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}.
\end{aligned}$$

The proof of Theorem 2.6 is complete. □

Corollary 2.7. *Under conditions of Theorem 2.6, if* $p=\ell\le\min\bigl\{q,\frac{q}{q-1}\bigr\}$, *then we have*

$$\begin{aligned}&\frac{1}{b-a}\int_a^b f(x)g(x)\,dx\\&\le\frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}\\&\times\bigl[\min\bigl\{M\bigl(a,b;f^p,g^{q-p(q-1)}\bigr),M\bigl(b,a;f^p,g^{q-p(q-1)}\bigr)\bigr\}\bigr]^{1/q}\\&\times\bigl[\min\bigl\{M\bigl(a,b;f^{(q-p)/(q-1)},g^p\bigr),M\bigl(b,a;f^{(q-p)/(q-1)},g^p\bigr)\bigr\}\bigr]^{1-1/q}.\end{aligned}$$

Corollary 2.8. *Under conditions of Theorem 2.6, when* $\alpha_1=\alpha_2=m_1=m_2=1$, *we have*

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,dx&\le\frac{1}{6}\bigl[2f^p(a)g^{q-\ell(q-1)}(a)+f^p(a)g^{q-\ell(q-1)}(b)\\&+f^p(b)g^{q-\ell(q-1)}(a)+2f^p(b)g^{q-\ell(q-1)}(b)\bigr]^{1/q}\bigl[2f^{(q-p)/(q-1)}(a)g^{\ell}(a)\\+f^{(q-p)/(q-1)}(a)g^{\ell}(b)&+f^{(q-p)/(q-1)}(b)g^{\ell}(a)+2f^{(q-p)/(q-1)}(b)g^{\ell}(b)\bigr]^{1-1/q}.\end{aligned}$$

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References

[1] R.-F. Bai, F. Qi, and B.-Y. Xi, *Hermite-Hadamard type inequalities for the m- and* (α,m)*-logarithmically convex functions*, Filomat, **27.1** (2013), 1–7; `http://dx.doi.org/10.2298/FIL1301001B`.

[2] M. K. Bakula, M. E. Özdemir, and J. Pečarić, *Hadamard type inequalities for m-convex and* (α,m)*-convex functions*, J. Inequal. Pure Appl. Math. **9.4** (2008), Article 96, 12 pages; `http://www.emis.de/journals/JIPAM/article1032.html`.

[3] S. S. Dragomir and G. Toader, *Some inequalities for m-convex functions*, Studia Univ. Babeş-Bolyai Math., **38.1** (1993), 21–28.

[4] V. G. Miheşan, *A generalization of the convexity*, Seminar on Functional Equations, Approx. Convex, Cluj-Napoca, 1993. (Romania)

[5] B. G. Pachpatte, *On some inequalities for convex functions*, RGMIA Res. Rep. Coll. **6** (2003), Article 1; `http://rgmia.org/v6(E).php`.

[6] Y. Shuang, H.-P. Yin, and F. Qi, *Hermite-Hadamard type integral inequalities for geometric-arithmetically s-convex functions*, Analysis (Munich) **33.2** (2013), 197–208; `http://dx.doi.org/10.1524/anly.2013.1192`.

[7] G. Toader, *Some generalizations of the convexity*, Proceedings of the Colloquium on Approximation and Optimization, Univ. Cluj-Napoca, Cluj, 1985, 329–338.
[8] B.-Y. Xi and F. Qi, *Hermite-Hadamard type inequalities for functions whose derivatives are of convexities*, Nonlinear Funct. Anal. Appl., **18.2** (2013), 163–176.
[9] B.-Y. Xi and F. Qi, *Some Hermite-Hadamard type inequalities for differentiable convex functions and applications*, Hacet. J. Math. Stat., **42.3** (2013), 243–257.
[10] B.-Y. Xi and F. Qi, *Some inequalities of Hermite-Hadamard type for h-convex functions*, Adv. Inequal. Appl., **2.1** (2013), 1–15.
[11] B.-Y. Xi, Y. Wang, and F. Qi, *Some integral inequalities of Hermite-Hadamard type for extended (s,m)-convex functions*, Transylv. J. Math. Mechanics, **5.1** (2013), 69–84.
[12] T.-Y. Zhang, A.-P. Ji, and F. Qi, *Some inequalities of Hermite-Hadamard type for GA-convex functions with applications to means*, Matematiche (Catania), **68.1** (2013), 229–239; http://dx.doi.org/10.4418/2013.68.1.17.

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