PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than August 1, 2000, although solutions received after that date will also be considered until the time when a solution is published.

129. *Proposed by Kenneth B. Davenport, 301 Morea Road, Box 491, Frackville, Pennsylvania.*

Let $k \geq 0$ and $i \geq 1$ be integers. Prove that

$$\sum_j \left\langle {k \atop j} \right\rangle \binom{k+1+i-j}{k+1} = \sum_{m=1}^{i} m^k,$$

where

$$\left\langle {k \atop j} \right\rangle$$

denotes an Eulerian number.

130. *Proposed by Joseph Wiener and William Heller, University of Texas-Pan American, Edinburg, Texas.*

Show that for any $b > 1$, the function

$$f(x) = \big(x^2 + (1-b)\big)e^x + bx$$

has exactly one zero for $x \geq 0$.

131. *Proposed by Kenneth B. Davenport, 301 Morea Road, Box 491, Frackville, Pennsylvania.*

Show that if

$$A = \sum_{n=0}^{\infty}\left(\frac{1}{9n+1} - \frac{1}{9n+4}\right), \qquad B = \sum_{n=0}^{\infty}\left(\frac{1}{9n+5} - \frac{1}{9n+8}\right),$$

$$C = \sum_{n=0}^{\infty}\left(\frac{1}{9n+2} - \frac{1}{9n+5}\right), \qquad D = \sum_{n=0}^{\infty}\left(\frac{1}{9n+4} - \frac{1}{9n+7}\right),$$

then $A + B = (C + D)\alpha$, where $\alpha = 2\cos(\pi/9)$.

132. *Proposed by Don Redmond, Southern Illinois University, Carbondale, Illinois.*

Let F_n denote the nth Fibonacci number. That is, $F_0 = 0$, $F_1 = 1$ and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$. In 1883 Cesaro showed that

$$\sum_{k=0}^{n}\binom{n}{k}F_k = F_{2n} \quad \text{and} \quad \sum_{k=0}^{n}\binom{n}{k}2^k F_k = F_{3n}.$$

Prove the following generalization of Cesaro's result.

Let r and s be roots of the quadratic equation

$$x^2 - ax - b = 0. \tag{1}$$

Define the two sequences $\{P_n\}$ and $\{Q_n\}$ by

$$Q_n = \frac{r^n - s^n}{r - s} \quad \text{and} \quad P_n = cr^n + ds^n,$$

where c and d are constants. If $j \geq 2$, then

$$\sum_{k=0}^{n}\binom{n}{k}(bQ_{j-1})^{n-k}Q_j^k P_k = P_{jn}.$$