PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than April 1, 1998, although solutions received after that date will also be considered until the time when a solution is published.

105. *Proposed by Kenneth Davenport, P. O. Box 99901, Pittsburgh, Pennsylvania.*

Evaluate the series

$$\frac{3}{1}+\frac{1}{3}-\frac{1}{6}-\frac{1}{10}+\cdots$$

where the denominators are the triangular numbers and every two terms the signs alternate, i.e. $+, +, -, -, +, +$, etc.

106. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Show that

$$\frac{1}{2}\sum_{m=0}^{n}\binom{n}{m}\frac{1}{(n-m+1)(m+1)}=\frac{2^{n+1}-1}{(n+2)(n+1)}.$$

107. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

Prove, if $p = 8k \pm 3$ is a prime for $k \geq 1$ and

$$a^2 + (p-2)b^2 \equiv 0 \pmod{p},$$

then $a \equiv 0 \pmod{p}$ and $b \equiv 0 \pmod{p}$.

108. *Proposed by Joseph B. Dence, University of Missouri-St. Louis, St. Louis, Missouri.*

A positive integer d is called a unitary divisor of a positive integer n, written $d \,||\, n$, if d and n/d are relatively prime. We define two unitary arithmetic functions by analogy to their standard counterparts:

A unitary Möbius function $\mu^*(n)$:

$$\sum_{d \,||\, n} \mu^*(d) = \begin{cases} 1, & \text{for } n = 1; \\ 0, & \text{for } n > 1. \end{cases}$$

A unitary Euler phi-function $\phi^*(n)$:

$$\phi^*(n) = \sum_{d \,||\, n} \mu^*(d) \frac{n}{d}.$$

When $n > 2$, $\phi(n)$ is always even; this is not true of $\phi^*(n)$. Determine how many known odd primes are in the range of the function $\phi^*(n)$.