PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to ccooper@cmsuvmb.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 1, 1998, although solutions received after that date will also be considered until the time when a solution is published.

101. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Find the coefficient of x^{1997} (in closed form) in the expansion of

$$\sqrt{2x^2 - 3x^3}.$$

102. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

(a) Prove if p is a positive prime of the form $p = 4k + 1$, then k and $-k$ are quadratic residues.

(b) Let p be a positive prime of the form $p = 4k + 3$.

i. Prove k is a quadratic residue if and only if $-k$ is a quadratic non-residue.

ii. Prove k is a quadratic residue if $p \equiv 1 \pmod 3$ and k is a quadratic non-residue if $p \equiv 2 \pmod 3$.

103. *Proposed by Thomas Dence, University of Georgia, Athens, Georgia and Joseph B. Dence, University of Missouri-St. Louis, St. Louis, Missouri.*

Find a closed form for the sum

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{1}{7}-\frac{1}{8}+\frac{1}{10}-\frac{1}{11}+\frac{1}{13}-\frac{1}{14}+\cdots .$$

104. *Proposed by Kenneth Davenport, P. O. Box 99901, Pittsburgh, Pennsylvania.*

Show that

$$1\cdot\sin\frac{\pi}{2n}+3\cdot\sin\frac{3\pi}{2n}+5\cdot\sin\frac{5\pi}{2n}+\cdots+(2n-1)\sin\frac{(2n-1)\pi}{2n}=n\csc\frac{\pi}{2n}.$$