PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than July 31, 1994, although solutions received after that date will also be considered until the time when a solution is published.

61. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Let n be a positive integer greater than one. Prove that

$$(n-1)!\left(\sum_{k=1}^{n} k(k!)^{\frac{1}{k}}\right)\left(\prod_{k=1}^{n}(k!)^{\frac{1}{k}}\right) < 2^{n(n+3)/2}.$$

62. *Proposed by Jayanthi Ganapathy, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

Express explicitly in terms of x, all those functions $f(x)$ with domain (M,∞) for some real number M, that have the following properties.

(a) f is increasing and differentiable on (M,∞).

(b) $0 < f'(x) < 1$ whenever $f(x) > 0$ and $f'(x) > 1$ whenever $f(x) < 0$.

(c)

$$e^{f(x)} - f'(x) = -\left(\frac{1}{e^{f(x)}} - \frac{1}{f'(x)}\right),$$

for all x in (M,∞).

63. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Find the roots of the equation

$$8x^6 - 42ix^5 - 21x^4 - 84ix^3 - 21x^2 - 42ix + 8 = 0,$$

where $i^2 = -1$.

64. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

For n a positive integer, let M_n denote the $n \times n$ matrix (a_{ij}) where $a_{ij} = i + j$. Is there a simple formula for perm (M_n)?