

## PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvm.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (\*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than February 28, 1994, although solutions received after that date will also be considered until the time when a solution is published.

**57.** *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Let  $s$  and  $k$  be positive integers. Evaluate

$$\lim_{n \rightarrow \infty} \prod_{i=1}^k \sum_{j=1}^n j^i \left( \frac{s}{n} \right)^{i+1}.$$

**58.** *Proposed by Kandasamy Muthuvel, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

Let  $G$  be a proper subgroup of  $\mathbb{R}$ , the reals under addition. Prove that  $\mathbb{R}$  and the complement of  $G$  have the same cardinality.

**59.** *Proposed by Ollie Nanyes, Bradley University, Peoria, Illinois.*

Find a topology  $\tau_1$  for the real line  $\mathbb{R}^1$  such that:

- 1)  $(\mathbb{R}^1, \tau_1)$  is a second countable, metrizable space and
- 2) there is a homeomorphism

$$f: (\mathbb{R}^1, \tau_1) \rightarrow (\mathbb{R}^2, \tau_2),$$

where  $\tau_2$  is the product topology  $\tau_1 \times \tau_1$ .

**60.** *Proposed by Alvin Tinsley, Central Missouri State University, Warrensburg, Missouri.*

Suppose a unit square has its left-hand corner at the origin and its sides along the  $x$ - &  $y$ -axes. Initially, place the base of an equilateral triangle with unit sides on the  $x$ -axis between 0 and 1. Slide the triangle to the left and up, always keeping the two vertices of the base in contact with the  $x$ - &  $y$ -axes until the base of the equilateral triangle is on the  $y$ -axis between 0 and 1. What is the equation of the locus of points determined by the third vertex of the triangle?