A GEOMETRIC APPROACH TO A TRIGONOMETRIC INTEGRAL

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An example given to a second semester calculus class consisted of evaluating

$$\int_0^{\pi} \sin^2 x \; dx \; .$$

The standard ploy used to determine this integral is to first change the integrand by using the trigonometric identity

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \; . \tag{1}$$

After carrying out the details to obtain the answer, a student with an apparent phobia for trigonometry inquired as to whether the solution could be obtained without using the half-angle formula (1). The following alternate method yielded an acceptable approach.

By considering the areas of the regions shaded in figures 1 and 2, it is clear that

$$\int_0^{\pi} \sin^2 x \; dx = \int_0^{\pi} \cos^2 x \; dx \; . \tag{2}$$

Substituting (2) into the left-hand side of the identity

$$\int_0^{\pi} (\sin^2 x + cos^2 x) \; dx = \int_0^{\pi} 1 \; dx = \pi$$

gives

$$2\int_0^{\pi} \sin^2 x \; dx = \pi \; ,$$

and hence the desired result after dividing by 2.

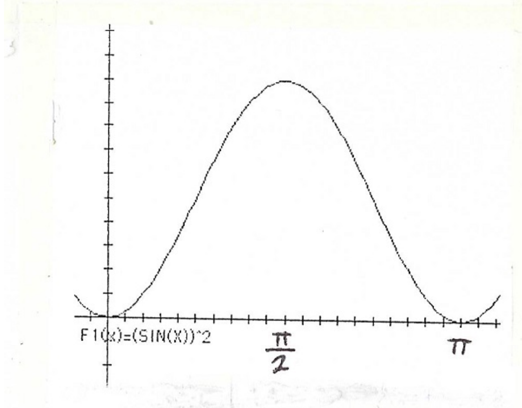

Figure 1.

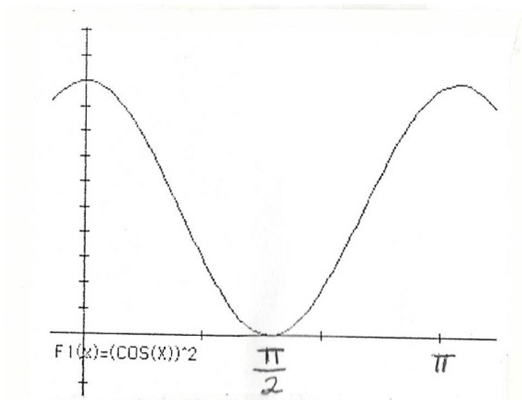

Figure 2.