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A theorem in the theory of definition

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The well-known theorem of Beth on definability can be extended in different directions. One was pursued by Svenonius [**1**], another by Kueker [**2**]. By using the extended form of preservation theorems developed in Motohashi [**5**], Weglorz [**4**], we shall get an extension of Beth's theorem of a new kind in this paper.

§ 0. Preliminaries

We shall use the ordinary set-theoretical and model-theoretical notations (see [**3**], [**5**]). In this paper, we shall be concerned with the first order predicate calculus with equality $\doteq$, (abbr. by f.p.c.), $L, L', \cdots$, will be used to denote f.p.c. For L, $M(L)$ is the class of all the first order structures related to L.

Let $M=\bigcup_L M(L)$. For L, L', $L\subset L'$, $L\cap L'$ have obvious meanings. L_0 denotes the f.p.c. without logical constants. Therefore $M(L_0)$ is the class of all non empty sets. Let $L\subset L'$, $\mathfrak{A}\in M(L')$, then $\mathfrak{A}\lceil L$ means the reduct of $\mathfrak{A}$ to L, $|\mathfrak{A}|$ is the universe of $\mathfrak{A}$, and $\bar{\bar{\mathfrak{A}}}=|\bar{\bar{\mathfrak{A}}}|$. If $\gamma_1, \gamma_2, \cdots, \gamma_m$ are non logical constants, then $L(\gamma_1, \cdots, \gamma_m)$ is the f.p.c. having $\gamma_1, \cdots, \gamma_m$ as non logical constants in addition to those of L. For $\mathfrak{A}, \mathfrak{B}\in M(L)$ and $f\in|\mathfrak{B}|^{|\mathfrak{A}|}$, f is said to be an *embedding* of $\mathfrak{A}$ to $\mathfrak{B}$ if f is an injection and the image of $\mathfrak{A}$ by f is the substructure of $\mathfrak{B}$. For $\mathfrak{A}\in M(L)$, $L(\mathfrak{A})$ means the diagram language of $\mathfrak{A}$.

We assume that the reader is familiar with the notion of special models (see Morley-Vaught [**3**]).

§ 1. Main theorem

An operation $\mathcal{M}$ defined on M^2 is said to be a *morphism on models* (*m.o.m.*) if $\mathcal{M}(\mathfrak{A}, \mathfrak{B})\subset|\mathfrak{B}|^{|\mathfrak{A}|}$ for $\mathfrak{A}, \mathfrak{B}\in M$.

For *m.o.m.* $\mathcal{M}$ and L, we define $\Delta_L(\mathcal{M})$ by the set of all formulas $F(v_0, v_1, \cdots, v_n)\in\mathfrak{F}(L)$ such that $\mathfrak{A}\models F[a_0, a_1, \cdots, a_n]$ implies $\mathfrak{B}\models F[f(a_0), f(a_1), \cdots, f(a_n)]$, for any $\mathfrak{A}, \mathfrak{B}\in M(L)$, $f\in\mathcal{M}(\mathfrak{A}, \mathfrak{B})$, $\langle a_0, a_1, \cdots, a_n\rangle\in|\mathfrak{A}|^{n+1}$.

DEFINITION. Let $\mathcal{M}$ be a *m.o.m.*

$\mathcal{M}$ is *natural* if it satisfies the following conditions:

(a) $\mathcal{M}(\mathfrak{A}, \mathfrak{B}) \subset \mathcal{M}(\mathfrak{A}\upharpoonright L, \mathfrak{B}\upharpoonright L)$ for any $\mathfrak{A}, \mathfrak{B} \in M(L')$, $L \subset L'$.

(b) $t_1 \doteq t_2 \in \Delta_L(\mathcal{M})$ for any terms t_1, t_2 in L.

(c) For any f.p.c. L, $\mathfrak{A}, \mathfrak{B} \in M(L)$ and $f \in \mathcal{M}(|\mathfrak{A}|, |\mathfrak{B}|)$, we have $f \in \mathcal{M}(\mathfrak{A}, \mathfrak{B})$ if and only if $(\mathfrak{A}, a)_{a\in|\mathfrak{A}|} \cap \Delta_{L(\mathfrak{A})}(\mathcal{M}) \subset \mathrm{Th}\,(\mathfrak{B}, f(a))_{a\in|\mathfrak{A}|}$

(d) For any f.p.c. L and $\mathfrak{A}, \mathfrak{B} \in M(L)$ such that $\mathfrak{A}$ and $\mathfrak{B}$ are special and one of the conditions: $\bar{\bar{\mathfrak{A}}} = \bar{\bar{\mathfrak{B}}}$ or $\bar{\bar{\mathfrak{A}}} < \omega$ or $\bar{\bar{\mathfrak{B}}} < \omega$, is satisfied, we have

$$\mathcal{M}(\mathfrak{A}, \mathfrak{B}) \neq \phi \qquad \text{if and only if } \mathrm{Th}\,\mathfrak{A} \cap \Delta_L(\mathcal{M}) \subset \mathrm{Th}\,\mathfrak{B}\,.$$

Define $\mathcal{M}_i$, $\mathcal{M}_h$, and $\mathcal{M}_e$ as follows:

For $\mathfrak{A}, \mathfrak{B} \in M$, $\mathcal{M}_i(\mathfrak{A}, \mathfrak{B})$ (resp. $\mathcal{M}_h(\mathfrak{A}, \mathfrak{B})$, $\mathcal{M}_e(\mathfrak{A}, \mathfrak{B})$) is the set of all the isomorphisms (resp. homomorphisms, embeddings) of $\mathfrak{A}$ to $\mathfrak{B}$ if $\mathfrak{A}, \mathfrak{B} \in M(L)$ for some L, and $\mathcal{M}_i(\mathfrak{A}, \mathfrak{B}) = \phi$ (resp. $\mathcal{M}_h(\mathfrak{A}, \mathfrak{B}) = \phi$, $\mathcal{M}_e(\mathfrak{A}, \mathfrak{B}) = \phi$) otherwise.

Then, $\mathcal{M}_i$, $\mathcal{M}_h$ and $\mathcal{M}_e$ are examples of natural morphisms on models and $\Delta_L(\mathcal{M}_i) = \mathfrak{F}(L)$, $\Delta_L(\mathcal{M}_h) =$ the set of formulas in L equivalent to positive formulas and $\Delta_L(\mathcal{M}_e) =$ the set of formulas in L equivalent to existential formulas.

THEOREM. *Suppose $L \subset L'$, T is a theory in L' and $\mathcal{M}$ is natural.*

(I) *If $\mathcal{M}(\mathfrak{A}, \mathfrak{B}) = \mathcal{M}(\mathfrak{A}\upharpoonright L, \mathfrak{B}\upharpoonright L)$ for any models $\mathfrak{A}, \mathfrak{B}$ of T, then for any formula $F(v_0, v_1, \cdots, v_n) \in \Delta_{L'}(\mathcal{M})$, there is a formula $G(v_0, v_1, \cdots, v_n) \in \Delta_L(\mathcal{M})$ such that*

$$T \vdash (\forall v_0)(\forall v_1) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n))\,.$$

(II) *If $\mathcal{M}(\mathfrak{A}, \mathfrak{B}) = \mathcal{M}(\mathfrak{A}\upharpoonright L, \mathfrak{B}\upharpoonright L)$ for any models $\mathfrak{A}, \mathfrak{B}$ of T such that $\mathfrak{A} \cong \mathfrak{B}$, then for any formula $F(v_0, v_1, \cdots, v_n) \in \Delta_{L'}(\mathcal{M})$, there are $G_j(v_0, \cdots, v_n) \in \Delta_L(\mathcal{M})$, $j = 1, \cdots, m$ such that*

$$T \vdash \bigvee_{j=1}^{m} (\forall v_0) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G_j(v_0, \cdots, v_n))\,.$$

PROOF. (I) Suppose the hypothesis of (I) is satisfied. Let $F(v_0, \cdots, v_n) \in \Delta_{L'}(\mathcal{M})$.

Let Σ be the set of formulas $G(v_0, \cdots, v_n) \in \Delta_L(\mathcal{M})$ such that $T \vdash F(v_0, \cdots, v_n) \rightarrow G(v_0, \cdots, v_n)$.

Now, assume $T \cup \Sigma \cup \{\neg F(v_0, \cdots, v_n)\}$ is consistent.

Then there is a model $\mathfrak{B}'$ of T and $\langle b_0, \cdots, b_n\rangle \in |\mathfrak{B}'|^{n+1}$ such that $\mathfrak{B}' \models \neg F[b_0, \cdots, b_n]$ and $\mathfrak{B}' \models G[b_0, \cdots, b_n]$ for any $G(v_0, \cdots, v_n) \in \Sigma$. Let Σ' be the set of formulas $H(v_0, \cdots, v_n)$ such that $\mathfrak{B}' \models H[b_0, \cdots, b_n]$ and $\neg H(v_0, \cdots, v_n) \in \Delta_L(\mathcal{M})$.

Then by the definition of Σ and Σ', $T \cup \Sigma' \cup \{F(v_0, \cdots, v_n)\}$ is consistent. So, there is a model $\mathfrak{A}'$ of T and $\langle a_0, \cdots, a_n\rangle \in |\mathfrak{A}'|^{n+1}$ such that $\mathfrak{A}' \models F[a_0, \cdots, a_n]$ and $\mathfrak{A}' \models H[a_0, \cdots, a_n]$ for any $H(v_0, \cdots, v_n) \in \Sigma'$. Then $\mathrm{Th}\,(\mathfrak{A}'\upharpoonright L, a_0, \cdots,$

$a_n) \cap \Delta_{L(a_0,\cdots,a_n)}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B}' \upharpoonright L, b_0, \cdots, b_n)$.

Then by [3], there are two special models $\mathfrak{A}$, $\mathfrak{B}$ of T such that $\bar{\bar{\mathfrak{A}}} = \bar{\bar{\mathfrak{B}}}$ or $\bar{\bar{\mathfrak{A}}} < \omega$ or $\bar{\bar{\mathfrak{B}}} < \omega$ and $\mathfrak{A} > \mathfrak{A}'$, $\mathfrak{B} > \mathfrak{B}'$.

Therefore $\mathrm{Th}(\mathfrak{A} \upharpoonright L, a_0, \cdots, a_n) \cap \Delta_{L(a_0,\cdots,a_n)}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B} \upharpoonright L, b_0, \cdots, b_n)$.

By (d), we get $\mathcal{M}((\mathfrak{A} \upharpoonright L, a_0, \cdots, a_n), (\mathfrak{B} \upharpoonright L, b_0, \cdots, b_n)) \neq \phi$.

Let $f \in \mathcal{M}((\mathfrak{A} \upharpoonright L, a_0, \cdots, a_n), (\mathfrak{B} \upharpoonright L, b_0, \cdots, b_n))$, then $f \in \mathcal{M}(\mathfrak{A} \upharpoonright L, \mathfrak{B} \upharpoonright L)$ and $f(a_i) = b_i$, $i = 0, \cdots, n$ by (a), (b). Since $\mathcal{M}(\mathfrak{A}, \mathfrak{B}) = \mathcal{M}(\mathfrak{A} \upharpoonright \mathrm{L}, \mathfrak{B} \upharpoonright L)$, we have $f \in \mathcal{M}(\mathfrak{A}, \mathfrak{B})$.

As we have $F(v_0, \cdots, v_n) \in \Delta_{L'}(\mathcal{M})$ and $\mathfrak{A} \models F[a_0, a_1, \cdots, a_n]$, we get $\mathfrak{B} \models F[f(a_0), f(a_1), \cdots, f(a_n)]$ by the definition of $\Delta_{L'}(\mathcal{M})$. So, we obtain $\mathfrak{B} \models F[b_0, b_1, \cdots, b_n]$. But this contradicts $\mathfrak{B} \models \neg F[b_0, b_1, \cdots, b_n]$.

Therefore $T \cup \Sigma \cup \{F(v_0, \cdots, v_n)\}$ is inconsistent.

By the compactness theorem and the definition of Σ, we get $G(v_0, \cdots, v_n) \in \Delta_L(\mathcal{M})$ such that

$$T \vdash (\forall v_0)(\forall v_1) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n)) .$$

(II) Suppose the hypothesis of (II) is satisfied. Let $F(v_0, v_1, \cdots, v_n) \in \Delta_L(\mathcal{M})$.

Let $\Sigma = \{\neg(\forall v_0) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n)) : G(v_0, \cdots, v_n) \in \Delta_L(\mathcal{M})\}$. Assume that $T \cup \Sigma$ is consistent.

Let $\mathfrak{C}$ be its model, and $T' = \mathrm{Th}\,\mathfrak{C}$. Let $\mathfrak{A}$, $\mathfrak{B}$ be arbitrary models of T' and $f \in \mathcal{M}(\mathfrak{A} \upharpoonright L, \mathfrak{B} \upharpoonright L)$. Then by (a), we have $f \in \mathcal{M}(|\mathfrak{A}|, |\mathfrak{B}|)$. Hence by (c), $\mathrm{Th}(\mathfrak{A} \upharpoonright L, a)_{a \in |\mathfrak{A}|} \cap \Delta_{L(\mathfrak{A})}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B} \upharpoonright L, f(a))_{a \in |\mathfrak{A}|}$. By [3], we get two special models $\mathfrak{A}_1$, $\mathfrak{B}_1$ of T' such that $\mathfrak{A}_1 > \mathfrak{A}$, $\mathfrak{B}_1 > \mathfrak{B}$ and $\bar{\bar{\mathfrak{A}}}_1 = \bar{\bar{\mathfrak{B}}}_1$, or $\bar{\bar{\mathfrak{A}}}_1 < \omega$ or $\bar{\bar{\mathfrak{B}}} < \omega$. Hence, $\mathrm{Th}(\mathfrak{A}_1 \upharpoonright L, a)_{a \in |\mathfrak{A}|} \cap \Delta_{L(\mathfrak{A})}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B}_1 \upharpoonright L, f(a))_{a \in |\mathfrak{A}|}$ and $\mathfrak{A}_1 \cong \mathfrak{B}_1$ because $\mathfrak{A}_1 \equiv \mathfrak{B}_1$.

By (d), $\mathcal{M}((\mathfrak{A}_1 \upharpoonright L, a)_{a \in |\mathfrak{A}|}, (\mathfrak{B}_1 \upharpoonright L, f(a))_{a \in |\mathfrak{A}|}) \neq \phi$. Let $g \in \mathcal{M}((\mathfrak{A}_1 \upharpoonright L, a)_{a \in |\mathfrak{A}|}, (\mathfrak{B}_1 \upharpoonright L, f(a))_{a \in |\mathfrak{A}|})$. Then by (a), (b), we get $g \upharpoonright |\mathfrak{A}| = f$ and $g \in \mathcal{M}(\mathfrak{A}_1 \upharpoonright L, \mathfrak{B}_1 \upharpoonright L)$.

By the hypothesis of (II), $\mathcal{M}(\mathfrak{A}_1 \upharpoonright L, \mathfrak{B}_1 \upharpoonright L) = \mathcal{M}(\mathfrak{A}_1, \mathfrak{B}_1)$. Hence $g \in \mathcal{M}(\mathfrak{A}_1, \mathfrak{B}_1)$.

By (c) $\mathrm{Th}(\mathfrak{A}_1, a)_{a \in |\mathfrak{A}_1|} \cap \Delta_{L(\mathfrak{A}_1)}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B}_1, g(a))_{a \in (\mathfrak{A}_1)}$.

Therefore $\mathrm{Th}(\mathfrak{A}, a)_{a \in |\mathfrak{A}|} \cap \Delta_{L(\mathfrak{A})}(\mathcal{M}) \subset \mathrm{Th}(\mathfrak{B}, f(a))_{a \in |\mathfrak{A}|}$.

By (c), $f \in \mathcal{M}(\mathfrak{A}, \mathfrak{B})$. We conclude $\mathcal{M}(\mathfrak{A} \upharpoonright L, \mathfrak{B} \upharpoonright L) = \mathcal{M}(\mathfrak{A}, \mathfrak{B})$ for any models $\mathfrak{A}$, $\mathfrak{B}$ of T'.

By (I) there is a $G(v_0, \cdots, v_n)$ in $\Delta_L(\mathcal{M})$ such that $T' \vdash (\forall v_0) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n))$.

Hence $\mathfrak{C} \models (\forall v_0) \cdots (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n))$. But this contradicts the fact that $\mathfrak{C}$ is a model of Σ. Hence $T \cup \Sigma$ is inconsistent.

By the compactness theorem, there are $G_j(v_0, v_1, \cdots, v_n) \in \Delta_L(\mathcal{M})$, $j = 1, \cdots, m$ such that $T \vdash \bigvee_{j=1}^{m} (\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow G_j(v_0, \cdots, v_n))$. q. e. d.

REMARK. The condition (c) is not necessary in the proof of (I).

§ 2. Some corollaries

In this section, we assume that P is an $(n+1)$-ary new predicate symbol which is not in L, and we set $L'=L(P)$. Let T be a theory in L'. Then by applying the theorem (I), (II) to $\mathcal{M}_i$, $\mathcal{M}_h$, $\mathcal{M}_e$, we can get the following six corollaries.

COROLLARY 1 (Beth's theorem). *The following three conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T, $\mathfrak{A}\upharpoonright L=\mathfrak{B}\upharpoonright L$ *implies* $\mathfrak{A}=\mathfrak{B}$.

(ii) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T, *if* f *is an isomorphism of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is an isomorphism of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(iii) *For some formula* $G(v_0, \cdots, v_n)$ *in* L, *we have*

$$T\vdash(\forall v_0)\cdots(\forall v_n)(P(v_0, \cdots, v_n)\leftrightarrow G(v_0, \cdots, v_n)).$$

COROLLARY 2 (Svenonius' theorem). *The following three conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T *such that* $\mathfrak{A}\cong\mathfrak{B}$, $\mathfrak{A}\upharpoonright L=\mathfrak{B}\upharpoonright L$ *implies* $\mathfrak{A}=\mathfrak{B}$.

(ii) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T *such that* $\mathfrak{A}\cong\mathfrak{B}$, *if* f *is an isomorphism of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is an isomorphism of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(iii) *For some formulas* $G_j(v_0, v_1, \cdots, v_n)$, $j=1, 2, \cdots, m$ *in* L

$$T\vdash\bigvee_{j=1}^{m}(\forall v_0)(\forall v_1)\cdots(\forall v_n)(P(v_0, v_1, \cdots, v_n)\leftrightarrow G_j(v_0, v_1, \cdots, v_n)).$$

COROLLARY 3. *The following two conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T, *if* f *is a homomorphism of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is a homomorphism of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(ii) *For some positive formula* $G(v_0, \cdots, v_n)$ *in* L,

$$T\vdash(\forall v_0)\cdots(\forall v_n)(P(v_0, \cdots, v_n)\leftrightarrow G(v_0, \cdots, v_n)).$$

COROLLARY 4. *The following two conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T *such that* $\mathfrak{A}\cong\mathfrak{B}$, *if* f *is a homomorphism of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is a homomorphism of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(ii) *For some positive formulas* $G_j(v_0, \cdots, v_n)$, $j=1, \cdots, m$ *in* L,

$$T\vdash\bigvee_{j=1}^{m}(\forall v_0)\cdots(\forall v_n)(P(v_0, \cdots, v_n)\leftrightarrow G_j(v_0, \cdots, v_n)).$$

From now on, we assume that T is a universal theory in L' (i. e. the class of models of T is closed under substructure).

LEMMA. *Suppose* $F(v_0, \cdots, v_n)$ *is a universal formula in* L *and* $G(v_0, \cdots, v_n)$ *is an existential formula in* L. *If* $T\vdash(\forall v_0)\cdots(\forall v_n)(F(v_0, \cdots, v_n)\leftrightarrow G(v_0, \cdots, v_n))$, *then there is an open formula* $H(v_0, \cdots, v_n)$ *in* L *such that* $T\vdash(\forall v_0)\cdots$

$(\forall v_n)(F(v_0, \cdots, v_n) \leftrightarrow H(v_0, \cdots . v_n))$.

The proof of this lemma can be easily carried out by the standard method. (See [**6**]).

COROLLARY 5. *The following three conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T, $\mathfrak{A}\upharpoonright L \subset \mathfrak{B}\upharpoonright L$ *implies* $\mathfrak{A} \subset \mathfrak{B}$.

(ii) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T, *if* f *is an embedding of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is an embedding of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(iii) *For some open formula* $H(v_0, \cdots, v_n)$ *in* L,

$$T \vdash (\forall v_0) \cdots (\forall v_n)(P(v_0, \cdots, v_n) \leftrightarrow H(v_0, \cdots, v_n)).$$

PROOF. It is obvious that (iii) implies (ii), (ii) implies (i), and (i) implies (ii). Assume (ii). Then by the theorem (I), there are two existential formulas $F(v_0, \cdots, v_n)$, $G(v_0, \cdots, v_n)$ such that $T \vdash (\forall v_0) \cdots (\forall v_n)(P(v_0, \cdots, v_n) \leftrightarrow F(v_0, \cdots, v_n))$ and $T \vdash (\forall v_0) \cdots (\forall v_n)(\neg P(v_0, \cdots, v_n) \leftrightarrow G(v_0, \cdots, v_n))$.

Then by lemma, (iii) follows. q. e. d.

COROLLARY 6. *The following three conditions are equivalent:*

(i) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T *such that* $\mathfrak{A} \cong \mathfrak{B}$, $\mathfrak{A}\upharpoonright L \subset \mathfrak{B}\upharpoonright L$ *implies* $\mathfrak{A} \subset \mathfrak{B}$.

(ii) *For any models* $\mathfrak{A}$, $\mathfrak{B}$ *of* T *such that* $\mathfrak{A} \cong \mathfrak{B}$, *if* f *is an embedding of* $\mathfrak{A}\upharpoonright L$ *to* $\mathfrak{B}\upharpoonright L$, *then* f *is an embedding of* $\mathfrak{A}$ *to* $\mathfrak{B}$.

(iii) *For some open formulas* $H_j(v_0, \cdots, v_n)$, $j = 1, \cdots, m$ *in* L, *we have*

$$T \vdash \bigvee_{j=1}^{m} (\forall v_0) \cdots (\forall v_n)(P(v_0, \cdots, v_n) \leftrightarrow H_j(v_0, \cdots, v_n)).$$

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