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CORRECTION: ON THE HOLOMORPHIC AUTOMORPHISM GROUP OF A GENERALIZED HARTOGS TRIANGLE

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The purpose of this note is to correct the formulation of Theorem 1 in our previous paper [1]. In the proof of Theorem 1, Case (II), in [1] we made the erroneous claim in (3.7) that f_w is a linear automorphism of $\mathcal{E}_w^p$, provided that $p_1 \neq 1$. In fact, this only holds for $I \geq 2$. Thus we need to consider the case $I = 1$ separately. Because of this, we have to correct the formulation of Theorem 1 in [1] as follows:

THEOREM 1. *Let $\mathcal{H}_{\ell,m}^{p,q}$ be a generalized Hartogs triangle in $\mathbf{C}^{|\ell|} \times \mathbf{C}^{|m|}$ with $|m| = 1$. Then the holomorphic automorphism group $\mathrm{Aut}(\mathcal{H}_{\ell,m}^{p,q})$ consists of all transformations*

$$\Phi : (z_1, \dots, z_I, w) \longmapsto (\tilde{z}_1, \dots, \tilde{z}_I, \tilde{w})$$

of the following form:

Case I. $I = 1$.

(I.1) $q/p \in \mathbf{N}$: *In this case, putting $r = q/p$, we have*

$$\tilde{z}_1 = w^r H(z_1/w^r)\,, \quad \tilde{w} = Bw\,,$$

where $H \in \mathrm{Aut}(B^{\ell_1})$, where B^{ℓ_1} denotes the unit ball in $\mathbf{C}^{\ell_1}$, and $B \in \mathbf{C}$ with $|B| = 1$.

(I.2) $q/p \notin \mathbf{N}$: *In this case, we have*

$$\tilde{z}_1 = Az_1\,, \quad \tilde{w} = Bw$$

(*think of z_1 as column vectors*), *where $A \in U(\ell_1)$, the unitary group of degree ℓ_1, and $B \in \mathbf{C}$ with $|B| = 1$.*

Case II. $I \geq 2$.

(II.1) $p_1 = 1,\ q \in \mathbf{N}$: *In this case, we have*

$$\tilde{z}_1 = w^q H(z_1/w^q)\,, \quad \tilde{z}_i = \gamma_i(z_1/w^q) A_i z_{\sigma(i)} \ (2 \leq i \leq I)\,, \quad \tilde{w} = Bw$$

(*think of z_i as column vectors*), *where*

(1) $H \in \mathrm{Aut}(B^{\ell_1})$;

(2) *γ_i are nowhere vanishing holomorphic functions on B^{ℓ_1} defined by*

$$\gamma_i(z_1) = \left(\frac{1 - \|a\|^2}{(1 - \langle z_1, a \rangle)^2} \right)^{1/2p_i}, \quad a = H^{-1}(o) \in B^{\ell_1}\,,$$

where $\langle \cdot, \cdot \rangle$ denotes the standard Hermitian inner product on $\mathbf{C}^{\ell_1}$ and $o \in B^{\ell_1}$ is the origin of $\mathbf{C}^{\ell_1}$;

(3) $A_i \in U(\ell_i)$, *the unitary group of degree* ℓ_i, *and* $B \in \mathbf{C}$ *with* $|B| = 1$;

(4) σ *is a permutation of* $\{2, \dots, I\}$ *satisfying the following*: $\sigma(i) = s$ *can only happen when* $(\ell_i, p_i) = (\ell_s, p_s)$.

(II.2) $p_1 \neq 1$ *or* $q \notin \mathbf{N}$: *In this case, we have*

$$\tilde{z}_i = A_i z_{\sigma(i)} \ (1 \le i \le I), \quad \tilde{w} = Bw,$$

where $A_i \in U(\ell_i)$, $B \in \mathbf{C}$ *with* $|B| = 1$, *and* σ *is a permutation of* $\{1, \dots, I\}$ *satisfying the condition*: $\sigma(i) = s$ *can only happen when* $(\ell_i, p_i) = (\ell_s, p_s)$.

As is mentioned above, the assertion in the Case II of the theorem above is already verified in the proof of [1; Theorem 1]. Consider now the Case I, that is, $I = 1$. Then, putting $r = q/p$ as in the theorem, we have

$$\begin{aligned} \mathcal{H}^{p,q}_{\ell_1,1} &= \left\{(z_1, w) \in \mathbf{C}^{\ell_1} \times \mathbf{C};\ \|z_1\|^{2p} < |w|^{2q} < 1\right\} \\ &= \left\{(z_1, w) \in \mathbf{C}^{\ell_1} \times \mathbf{C};\ \|z_1\|^{2} < |w|^{2r} < 1\right\} = \mathcal{H}^{1,r}_{\ell_1,1} \end{aligned}$$

and hence $\mathrm{Aut}(\mathcal{H}^{p,q}_{\ell_1,1}) = \mathrm{Aut}(\mathcal{H}^{1,r}_{\ell_1,1})$ literally. Therefore, in the case where $I = 1$ and $r \in \mathbf{N}$, every element Φ of $\mathrm{Aut}(\mathcal{H}^{p,q}_{\ell_1,1})$ has the form as in (I.1), as is shown in the first half of Subsection 3.1 in [1]. Also, the verification of the assertion in (I.2) has been already done in the second half of Subsection 3.2 in [1]; thereby, the proof of Theorem 1 is completed.

References

[1] A. Kodama, On the holomorphic automorphism group of a generalized Hartogs triangle, Tohoku Math. J. 68 (2016), 29–45.

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