Real Analysis Exchange
Vol. 19(1), 1993/94, pp. 52–53

Krzysztof Ciesielski (kcies@wvnvms.wvnet.edu), Department of Mathematics, West Virginia University, Morgantown, WV 26506-6310

Lee Larson* (lmlars01@ulkyvx.louisville.edu), Department of Mathematics, University of Louisville, Louisville, KY 40292

UNIFORMLY ANTISYMMETRIC FUNCTIONS I

A function f is *weakly* continuous at x if there are sequences $a_n \nearrow 0$ and $b_n \searrow 0$ such that

$$\lim_{n\to\infty} f(x+a_n) = f(x) = \lim_{n\to\infty} f(x+b_n).$$

The following theorem is a well-known fact from cluster set theory.

Theorem 1 *If $f : \mathbb{R} \to \mathbb{R}$ is an arbitrary function, then f is weakly continuous on a cocountable set.*

Motivated by this, we look at the case of symmetric continuity instead.

Definition 1 *A function $f : \mathbb{R} \to \mathbb{R}$ is weakly symmetrically continuous at x if there is a sequence $h_n \searrow 0$ such that*

$$\lim_{n\to\infty} (f(x+a_n) - f(x-a_n)) = 0.$$

An easy density argument implies the following three propositions.

Theorem 2 *If f is measurable, then f is weakly symmetrically continuous almost everywhere.*

Theorem 3 *If f is a Baire function, then f is weakly symmetrically continuous on a residual set.*

Corollary 1 *Any function which fails to be weakly symmetrically continuous anywhere can be neither measurable nor Baire.*

Motivated by Theorem 2, the following question has been asked.

*Presenter

Question 1 (Evans-Larson 1984, Kostyrko 1991) *Does there exist a nowhere weakly symmetrically continuous function?*

The following theorem answers this question.

Theorem 4 *There is a function $f : \mathbb{R} \to \mathbb{N}$ which is nowhere weakly symmetrically continuous.*