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Dan Rinne, Department of Mathematics, California State University, San Bernardino, CA 92407

RECTANGULAR AND ITERATED CONVERGENCE OF MULTIPLE TRIGONOMETRIC SERIES

In a recent work, Tetunashvili [2] answered the uniqueness question for everywhere rectangularly convergent multiple trigonometric series. That is, if a series of the form $\sum a_{m_1,\ldots,m_d} e^{i(m_1x_1+\ldots+m_dx_d)}$ converges rectangularly to zero everywhere, then all coefficients are zero. By rectangular convergence we mean

$$\lim_{\min\{k_i\}\to\infty} \sum_{m_1=-k_1}^{k_1} \cdots \sum_{m_d=-k_d}^{k_d} a_{m_1,\ldots,m_d} e^{i(m_1x_1+\ldots+m_dx_d)}$$

exists. This question was also answered recently in [1]. Perhaps the most interesting part of Tetunashvili's solution is that his technique converts the rectangular convergence into iterated convergence assuming convergence everywhere. At that point, the uniqueness of the coefficients becomes an easy matter. We illustrate his proof in two dimensions.

References

[1] J. M. Ash, C. Freiling, and D. Rinne, *Uniqueness of rectangularly convergent trigonometric series*, Annals of Mathematics, **137** (1993), 145-166.

[2] SH. T. Tetunashvili, *On some multiple function series and the solution of the uniqueness problem for Pringsheim convergence of multiple trigonometric series*, Mat. Sb. **182** No. 8 (1991); English transl. in Math. USSR Sb. **73** No. 2 (1992), 517-534.