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BANACH–VALUED HENSTOCK INTEGRATION

A function f defined on $[a, b]$ and taking values in a Banach space X is said to be Henstock integrable on $[a, b]$ if there exists $z \in X$ with the following property: for every $\epsilon > 0$ there is a positive function δ on $[a, b]$ such that for any division $\mathcal{D}$ of $[a, b]$ given by

$$a = x_0 < x_1 < \ldots < x_n = b \text{ and } \{\xi_1, \xi_2, \ldots, \xi_n\},$$

satisfying

$$\xi_i \in [x_{i-1}, x_i] \subset (\xi_i - \delta(\xi_i), \xi_i + \delta(\xi_i)) \text{ for } i = 1, 2, \ldots, n,$$

we have

$$\|\sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}) - z\| < \epsilon.$$

The following results are given: (1) A Henstock integrable function need not be measurable; (2) The Henstock Lemma does not hold in general; and (3) A Bochner integrable function is Henstock integrable and satisfies the Henstock Lemma. A weak version of the Henstock Lemma is given. An example of a measurable function which is Henstock integrable but not Bochner integrable is given. The problem of extending the Henstock integral to Dunford and Pettis integrals and the extension of the definition of the Henstock integral to functions taking values in more general spaces are presented.