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Pamela B. Pierce, Department of Mathematics, Syracuse University, Syracuse, NY 13244

REGULATED FUNCTIONS WHOSE FOURIER SERIES CONVERGE FOR EVERY CHANGE OF VARIABLE

Let $I_{n,m}, m = 1, 2, \ldots, k_n$ be disjoint closed intervals such that for each n, $I_{n,m-1}$ is to the left of $I_{n,m}$. Given x, if for every $\epsilon > 0$ there exists N such that $I_{n,m} \subset (x, x+\epsilon)$ whenever $n > N$, then $\mathcal{I} = \{I_{n,m} : n = 1, 2, \ldots; m = 1, 2, \ldots, k_n\}$ is called a *right system* of intervals (at x). A left system is defined similarly. Let

$$\alpha_n(\mathcal{I}) = \sum_{i=1}^{k_n} \frac{f(I_{n,i})}{i} \qquad \textit{where} \qquad f([a,b]) = f(b) - f(a).$$

We present a proof of the following result:

Theorem 1 *If f is regulated, then $f \circ g$ has an everywhere convergent Fourier series for every homeomorphism g if and only if $\lim_{n\to\infty} \alpha_n(\mathcal{I}) = 0$ for every system $\mathcal{I}$ and for every x.*

This is a generalization of a theorem of Goffman and Waterman [1] which gave an analogous result for continuous functions. A principal tool in the proof was the following generalized Mean Value Theorem.

Theorem 2 *If f is defined on $[a,b]$ and $\mu \geq 0$ on $[a,b]$ then*

$$\int_a^b f(t)\mu(t)\,dt = c\int_a^b \mu(t)\,dt \qquad \textit{for some} \quad c \in [\inf_{[a,b]} f, \sup_{[a,b]} f].$$

Either there exists an x such that $f(x) = c$, or there exists an x in $[a,b]$ such that $\liminf_{t\to x} f(t) \leq c \leq \limsup_{t\to x} f(t)$.

References

[1] C. Goffman and D. Waterman, *Functions Whose Fourier Series Converge for Every Change of Variable,* Proc. Amer. Math. Soc. **19** (1968), 80-86.