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# TWO SUMMABILITY PROBLEMS – SPEEDS OF CONVERGENCE AND A RESULT OF POLLARD AND BUCK

**1.** During the early nineteenth century much effort was spent trying to find a "universal comparison test": *i.e.*, a sequence in $\ell^1$ that dominates every other member of $\ell^1$. The nonexistence of such a series converging at a minimal rate was demonstrated by Abel. In [1] J. Fridy proved that if $A$ is a regular matrix summability method and $t$ is a nonincreasing null sequence of reals, then there exists a null sequence $y$ such that

$$\rho_m t = o(\rho_m Ay).$$

Here, if $\lim_{k\to\infty} t_k = L$, $\rho_m t$ denotes the "maximum remaining distance" given by

$$\rho_m t = \max_{n>m} |t_n - L|.$$

In a joint work with Vanna Zanelli I have extended the above result of Fridy to the following.

**Theorem 1** *If $A$ is a regular matrix summability method and $t$ is a nonincreasing null sequence, then there exists a null sequence $y$ such that*

$$\lim_{m\to\infty} \frac{t_m}{(Ay)_m} = 0.$$

**2.** In 1940, Buck and Pollard proved that if $S = \{s_k\}$ is a $(C,1)$ summable sequence and $\sum \frac{s_k^2}{k^2} < \infty$ holds, then

$$\{x \in (0,1) : S(x) \text{ is } (C,1) \text{ summable to the } (C,1) \text{ limit of } S\}$$

has Lebesgue measure one. Here $S(x)$ is the subsequence of $S$ obtained by using the indices where the binary expansion of $x$ has the digit one (for example, if $x = \frac{0}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{0}{2^4} + \frac{1}{2^5} + \ldots$, then $S(x) = \{x_2, x_3, x_5, \ldots\}$).

Franz J. Schnitzer and I recently rediscovered: $\{x \in (0,1) : S(x) \text{ is } A \text{ summable}\}$ is a set of first Baire category if $A$ is a regular matrix summability method and

$S$ is a divergent sequence. In fact the above holds if $A$ is a non-Schur matrix with convergent columns. Also a rearrangement version of the last mentioned result holds.

In a joint paper Schnitzer and I have four results that extend the above. The following is a typical result.

**Theorem 2** *If $A$ is a non–Schur matrix with convergent columns and $S = \{s_k\}$ is a bounded divergent sequence, then the set $\{x \in (0,1) : S(x)$ is $B$ summable for some non-Schur matrix with convergent columns such that $\| A - B \| < \epsilon\}$ is of the first category for some $\epsilon > 0$.*

Here $\| C \| = \sup_p \left(\sum_{q=1}^{\infty} |c_{pq}|\right)$ if $C = (c_{pq})$.

## References

[1] J. A. Fridy, *Minimal rates of summability*, Can. J. Math., **XXX** (1978), 808-816.