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First Return Selections and Block Selections

In September 1989, the author suggested a topic for research to his student Gregory Rambousek. The subject was to investigate the behavior of types of selective derivatives and to find the connections between these and other known generalized derivatives.

Unfortunately, after completing preliminary work on the relationship between the definitions underlying the two selection types, Gregory became too ill to continue the project.

The purpose of this talk is to introduce the two selective derivatives and summarize Gregory's work.

[The project will be continued and the results presented in a later paper by Professors Evans, Humke and O'Malley.]

Definition 1. Let $\{x_n\}$ be any sequence of distinct points form $[0,1]$ which as a subset of $[0,1]$ is dense. For each closed nondegenerate subinterval, $[a,b]$ of $[0,1]$ let

$$s[a,b] = x_{n_1} \quad \text{for the first } n_1 \text{ satisfying } a < x_{n_1} < b.$$

The selection s thus defined is called the first return selection generated by the sequence $\{x_n\}$.

The label "first return" is motivated by concepts from dynamical systems. It is anticipated that one of the principal types of first return selections will be generated from trajectories of chaotic functions. For example: if $g(x) = 2x \bmod 1$ on $[0,1]$ then for almost all x_0 the sequence $x_n = g^n(x_0)$, $n = 0,1,2,\cdots$, can be used in the above definition.

Definition 2. A selection s, defined on all nondegenerate closed subintervals of $[0,1]$, is called a block selection if it satisfies the following condition:

Let $[a,b] \subseteq [0,1]$ be given and let $s[a,b] = p$. Then for all closed subintervals $[c,d]$ with $a \leq c < p < d \leq b$ we have $s[c,d] = p$. The block associated with $[a,b]$ is the rectangle $[a,p)\times(p,b]$.

Greg was able to show:

a) All first return selections are block selections.

b) There is a block selection which is not a first return selection for any sequence $\{x_n\}$.

c) For any block selection the collection of "maximal" blocks is countable, $\{M_n\}$.

d) A block selection is a first return selection if and only if the perimeter of M_n tends to 0 as n tends to $+\infty$.