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A COMPARISON OF THE JORDAN AND DINI TESTS: A CORRECTION

The purpose of [3] was to make precise the sense in which the Dini test was stronger than the Jordan test. It was shown that the Jordan test had strength 3 while the Dini test had strength ω_1. Unfortunately there is an elementary error in the proof of the latter result as given in [3 Prop.3]. In the proof it is claimed that the Zalwasser rank must be unbounded in ω_1 because D(T) is not an analytic subset of C(T). This claim is invalid because the set bD(T) of differentiable functions with bounded derivatives is not an analytic subset of C(T) (see[2]) but the Zalcwasser rank is bounded in ω_1 on bD(T). In fact the Zalcwasser rank of each function in bD(T) is 1. This is because a differentiable function with a bounded derivative has a uniformly covergent Fourier series.

So we must now rejustify our claim. The real truth of the matter depends on the following key result of Ajtai and Kechris [1]: There is no Borel subset B of C(T) such that $D(T) \subseteq B \subseteq FC$. Recall that FC was the set of continuous functions with every where convergent Fourier series. Now suppose that the Zalcwasser rank was bounded in ω_1 on D(T). Then we can find a countable ordinal α_0 such that $|f|_Z \leq \alpha_0$ for all f in D(T). So if we put $B = \{f \in C(T): |f|_Z \leq \alpha_0\}$, we have $D(T) \subseteq B \subseteq FC$. Since the Zalcwasser rank is a coanalytic norm, B is a Borel subset of C(T) (see [2]). This contradicts the above mentioned result of Ajtai and Kechris. Hence our claim is now really valid.

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