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More About 1-differentials on 1-cells

In [1],[2] the author defined differentials to be variational equivalence classes of summants for a Kurzweil-Henstock integral. The study of these differentials continues with the investigation of summable, damper-summable, archimedean, and weakly archimedean 1-differentials on a 1-cell K. Lebesgue-type convergence theorems hold for products of a differential with Borel functions under appropriate summability conditions. Two absolute continuity conditions involving indicator summants have lattice-theoretical formulations. The fundamental theorem of calculus is presented in terms of summant derivatives. Iterated integration-by-parts has a concise differential formulation: Given $n \geq 1$ and functions u, v of differentiability class $\mathcal{C}^n$ on K, let w be the continuous function defined by $w = \Sigma_{j=0}^{n} (-1)^j u^{(j)} v^{(n-j)}$. Then $dw = u\, dv^{(n)} + (-1)^n v\, du^{(n)}$. For v an appropriate polynomial of degree n this gives a generalized Taylor theorem.

[1] S. Leader, A concept of differential based on variational equivalence under generalized Riemann integration, Real Analysis Exchange 12 (1986-87), 144-175.

[2] ---------, What is a differential? A new answer from the generalized Riemann integral, Amer. Math. Monthly 93 (1986), 348-356.