NEW AND OLD RESULTS CONNECTED WITH HENSTOCK'S INTEGRALS

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From 1922 Beppo Levi[6] claimed that the Lebesgue integral could be defined with sums of Riemann-Cauchy type. The complete work was published in 1941 and is almost unknown.

The pseudo partitions used by Beppo Levi are coverings formed by finite or countable families of intervals. Each point of the domain of integration $[a,b]$ must be interior to some interval of the covering $\mathcal{D}$ such that $\forall x \in [a,b]$ there exist one interval $I \in \mathcal{D}$.

The integral is defined for bounded functions leaving open what to do in the case of no boundness.

E.J. McShane defined[2] a deterministic integral he used partitions which eventually do not cover a set of null measure. Both integrals are equivalent to the Lebesgue integral[2,4].

Beppo Levi's covering are too big, McShane covering's are too small. The finite partitions used by Henstock[5] to define the RC-integral are just right to define an integral which includes the Lebesgue integral and is more general.

In 1984 J.F. Colombeau[1] defined the generalized functions which include the Schwartz distributions and allows multiplication. Todorov[7] has given a non-standard model for Colombeau theory.

The punctual values defined for such generalized functions will be analized for certain distributions which have been expressed as NV-integrals $(NV)\int_{-\infty}^{\infty}\psi(x)dg(x)$ and related with the families of convergent factors $N(x;\pm h;t)$ and the function g used in those integrals.

Some intents to relate the product of distributions which are generalized functions with NV-integrals will be made via $(NV)\int_{-\infty}^{x}\psi(u)dg(u) = F_i(x)$ $i=1,2$, $F_1(x)$, $F_2(x) = F(x)$ and the use of generalized Russell derivatives.

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