P. Maritz, University of Stellenbosch, Stellenbosch 7600, South Africa

# OPEN AND IMAGE-OPEN MULTIFUNCTIONS

Henstock [5] and [6], Kurzweil [7] and Pfeffer [14], among others, employ positive functions in the integration theories that they consider. These positive functions are sometimes used in such a fashion that they may be replaced by image-open multifunctions; see the image-open multifunctions, called *gauges*, in McLeod [11] and McShane [12]. In [11] the Riemann integrable functions are characterised as those generalised Riemann integrable functions for which there is a gauge $\gamma$ on $[a,b]$ such that $\gamma(x) = (x - \delta, x + \delta)$, for some $\delta > 0$, and with the usual properties; such a gauge is lower-semi-continuous.

In some problems there is a great deal of freedom in defining a gauge to meet the requirements of the definition of the particular integral. In some cases a gauge $\gamma$ with the property that $\gamma(x)$ is of constant width for all (or for almost all) $x$ is sufficient; in other instances, a constant (or almost constant) gauge is chosen.

The purpose of this talk is to discuss some properties of multifunctions possessing some notions of openness.

The hypothesis of openness of a multifunction occurs very rarely in the literature. This is perhaps due to the fact that such multifunctions are, under fairly general conditions, "almost constant". Franklin [3] was the first to obtain results of this nature. Image-open multifunctions were discussed briefly by Choquet [2]. More recently, Münnich and Száz [13] established a theorem which not only improves a result of Stanojević [17], but which has among its applications improvements of results of Ponomarev [15] and, in particular, of Franklin's results on constant and "almost constant" multifunctions.

Graph-open multifunctions seem to occur in a natural way in mathematical economics. Gale and Mas-Colell [4] proved the existence of a Walrasian General Equilibrium by means of irreflexive preference mappings (multifunctions, here) with open graphs. (The augmented preference mappings in their paper must not have open graphs.) Mas-Colell [8] proved that if a multifunction has an open graph and its values are homeomorphically convex (an open star-shaped set is homeomorphically convex), then it has a continuous selection. In applications it is not uncommon to encounter fixed point theory of graph-open multifunctions, see [4] p. 10. In [1] Ceder and Levi gave an example of a lower-semi-continuous graph-open multifunction with no continuous selection and they also showed that certain graph-open multifunctions may have Borel 1 selections. Shafer and Sonnenschein [16] proved the existence of equilibrium in an abstract economy with preferences which may be both non-transitive and non-complete by using "preference correspondences" (that is, multifunctions) with open graphs. In [9] and [10] McClendon proved fixed point and selection theorems for subopen multifunctions and also for multifunctions with r-open graphs.

The fact that familiar topological concepts defined in terms of open covers (such as paracompactness, for instance) can be reformulated in terms of image-open multifunctions, suggests that image-open multifunctions should be investigated thoroughly with their intrinsic properties and applications as primary objectives. See also Yusufov [18] for recent results on open covering mappings.

This talk is based on a paper in which the following are the most important sections:

1. some counterexamples of results of Münnich and Száz [13]
2. the section on quasi-connectedness is designed in such a way that it

provides the necessary background needed for an investigation of

3. constant quasi-continuous multifunctions
4. multifunctions with non-mingled values
5. two selection results on image-open and graph-open multifunctions.

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