SOME APPLICATIONSOF A THEOREM OF MARCINKIEWICZ

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The theorem in question is stated in the classic book of Saks on page 253 and is

Theorem. If $f\colon [a,b] \to \mathbf{R}$ is measurable then f is Perron integrable if it has at least one continuous major function and are continuous minor function.

The only significant weakening of the continuity requirement is due to Sarkhel who showed that regulated would do and fairly simply example show that the theorem's false under any obvious weaker condition.

In another direction both Denjoy and McShane replaced upper and lower derivatives by upper and lower unilateral derivatives in the definitions of major and minor functions.

The talk presented a recent generalization by Bullen and Vyborny that dropped the continuity condition by spreading it over a family of major and minor functions.

Theorem. Hypothesis: $f\colon [a,b] \to \mathbf{R}$ is measurable; $\overline{A}, (\underline{A})$ is a nonempty family of major (minor) functions of f; for all $\epsilon > 0$ and for all x, $a \le x \le b$, there is a $\delta = \delta(x, \epsilon) > 0$ and an $M \in \overline{A}$, $m \in \underline{A}$ such that if I is a closed interval, $I \quad]x-\delta, x]$ or $I \quad]x, x+\delta[$ then

$- \epsilon \le m(I) \le M(I) \le \epsilon.$

Conclusion: f is Perron integrable.

This result, and its method of proof, implies the results of Sarkhel, Denjoy and McShane mentioned above, gives simple proofs a convergence theorem of Lee and Chew and of a very interesting result of Schurle.

Details will be published elsewhere.

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