

A FUNCTION IS PERRON INTEGRABLE IF IT HAS LOCALLY SMALL RIEMANN SUMS

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Using the generalized Riemann approach to the Perron integral we develop a new necessary and sufficient condition for Perron integrability. We assume familiarity with the notation and terminology of this approach as given, for example, in [P]. Then we say that a function f has locally small Riemann sums on an interval I if for each point b of I and each positive ϵ there is a positive function δ on I for which $|\sigma(f,P)| < \epsilon$ whenever P is a δ -fine Perron partition of an interval in $(b - \delta(b), b + \delta(b))$. Here $\sigma(f,P)$ denotes a Riemann sum for the function f . Our theorem then is as follows.

THEOREM. A measurable function f on an interval I is Perron integrable on I if and only if f has locally small Riemann sums on I .

The proof of sufficiency is accomplished by a careful investigation of the behavior of f on the set of points which have no neighborhoods on which f is Perron integrable. The main tool is judicious construction of δ -fine partitions with the use of the Baire Category Theorem.

We should note that in fact the proof applies to Perron-Stieltjes integrals with respect to a nondecreasing function α .

[P] W. Pfeffer, The Riemann-Stieltjes Approach to Integration, Technical Report 187, National Research Institute for Mathematical Sciences, Pretoria, South Africa.