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LIMITS AND SERIES OF EXTENDABLE CONNECTIVITY FUNCTIONS

A function from the unit interval I into $\mathbb{R}$ is *Darboux* if it maps connected sets to connected sets. A function $G : X \to \mathbb{R}$, where X is I or $I \times I$, is called a *connectivity* function if whenever C is a connected subset of X, then the graph of the restriction $G|C : C \to \mathbb{R}$ is a connected subset of $X \times \mathbb{R}$. A connectivity function $g : I \to \mathbb{R}$ is said to be *extendable* if there exists a connectivity function $G : I \times I \to \mathbb{R}$ for which $G(x, 0) = g(x)$ when $0 \leq x \leq 1$. The idea of g–negligible sets is useful. Given an extendable connectivity function $g : I \to I$, we say that a subset M of I is *g–negligible* if g may be redefined on M arbitrarily with values in I and still remain an extendable connectivity function.

For an arbitrary function $f : I \to I$, it is shown that

$$f = \lim_{n\to\infty} f_n \text{ and } f = \sum_{n=1}^{\infty} g_n$$

for some extendable connectivity functions $f_n : I \to I$ and $g_n : I \to \mathbb{R}$. An example is constructed of a sequence of extendable connectivity functions whose uniform limit is not Darboux. These results answer some questions of Richard Gibson.