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## **Carathéodory's outer measures: 80 years**

### **1 Constantin Carathéodory, 1873–1950**

“Constantin Carathéodory was a German mathematician of Greek descent.” I said this in [3]. But it is only a first approximation. [Approximations are appropriate for a conference in analysis, after all.] Actually, the facts are more complicated than that.

Certainly “Carathéodory” is a Greek name. The Carathéodory ancestors had lived in Constantinople (= Istanbul) since 1800 or earlier. Even when Greece obtained its independence from the Ottoman Empire in 1920, they remained in Istanbul.

Stephanos Carathéodory, the father of the mathematician, educated as a lawyer, was a diplomat on behalf of the Turkish government. He was stationed in Berlin in 1873, where his son Constantin was born on September 13. Two years after that, Stephanos was promoted to ambassador, and stationed in Brussels, where he remained for the next 30 years. Young Constantin grew up there, with Greek and French as his childhood languages.

When Carathéodory completed his schooling, he worked as an engineer in Paris, London, and Egypt. In 1900, at age 27, he decided to return to school to study mathematics. He chose Germany for his studies. He worked in Berlin and then Göttingen, under Schwarz and Hilbert. He completed his studies in 1904. His career as a mathematician was spent at German universities (Göttingen, Bonn, Hanover, Breslau, Göttingen again, Berlin, Munich) except for a short time in Greece from 1920 to 1924.

The above information is from [5].

### **2 Correspondence with Tibor Radó**

In my research on Carathéodory, I found an interesting item. The Hungarian mathematician Tibor Radó spent most of his career at The Ohio State University. After his death, his papers were deposited in the university archives. When the librarians cataloged these papers, they noted some correspondence

with Carathéodory. Then many years later, when I searched for information on Carathéodory, that entry was found by the computerized card catalog at OSU. I understand the thrill of the historian finding new information. There are four letters from Rado to Carathéodory, and four letters from Carathéodory to Rado, all in English. They illustrate some of Carathéodory's concerns during the last few years of his life.

A sample is reproduced on the two pages immediately following this report. The citation is [4].

### 3 The paper on outer measure

This year (1994) is the 80th anniversary of the publication of one of Carathéodory's revolutionary papers, "Über das lineare Maß von Punktmengen—eine Verallgemeinerung des Längenbegriffs" [1]. Carathéodory shows how the theory of outer measure may be used in the foundations of measure theory with considerable simplification over the previous methods of Lebesgue and Borel. Another advantage is that this theory may be used with measures (such as Hausdorff measures) where the conventional approach is impossible.

An English translation of this paper is in [3]. A few of my comments are also contained there.

### References

- [1] C. Carathéodory, "Über das lineare Maß von Punktmengen—eine Verallgemeinerung des Längenbegriffs". *Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse* 4 (1914) 494–426. Reprinted in [2], Band IV, pp. 249–277. English translation in [3], pp. 47–73.
- [2] C. Carathéodory, *Gesammelte Mathematische Schriften*. Bayerischen Akademie der Wissenschaften, C. H. Beck, Munich, 1954.
- [3] G. A. Edgar, *Classics on Fractals*. Addison-Wesley, 1993.
- [4] T. Radó, "Papers, 1921-1963". Ohio State University Archives 40:76.
- [5] E. Schmidt, "Constantin Carathéodory". In [2], Band V, pp. 411–419.

Munich December 16<sup>th</sup> 1948.  
Josephinum: Schöpfungstr. 16

Dear Radio;

Your charming letter reached me yesterday. I  
greatly appreciate your offer to send me a food parcel  
and thank you very much for it. But you need not be  
bothered about me because my daughter who is living partly  
in Tanganyika and partly in Greece is sending me  
everything I want. On the contrary I think that it would  
be very nice for you if you would send something to  
Prof. Pearson (München 27, Friedrich Henschelstr. 11)  
and to Prof. Tietze (München 22, Trautenwolfstr. 7)  
who both are in my judgement a little stumped.  
But I would suggest not to send CARE parcels, which  
during the last year have deteriorated in quality but  
to send smaller packages with things that you have  
chosen yourself.

Some weeks ago I had a long visit from  
Grego who had been nearly 3 months in Budapest  
and who told me very interesting details about Fujie  
and Riez and about the general situation of the

~~At the~~ Hungarian mathematicians. He had also visited Szeged where the scientific life seems to be thriving quite a bit.

I have been very, very ill during the last summer but now I have recovered all my forces. I am though still in hospital, but I hope that they will be able next month to make the operation I had to become again an active member of the society.

I would be greatly indebted to you if you would send me some reprints of your work about the area of surfaces. Some months ago I had a conversation about that topic with Besicovich and I have studied also some of his papers.

I have written lately a somewhat English Theory of Functions, which will be printed in Båle during the coming months.

With many greetings for Jonas & the New Year to you & your family

yours sincerely  
C. Carathéodory

P.S. I don't know if you are aware that I lost my dear wife 1 1/2 years ago. You can imagine how much this affected me.