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On Limits of Quasicontinuous, Simply Continuous and Cliquish Functions

Let X be a topological space and Y be a metric space (with metric d). A sequence (f_n), $f_n : X \to Y$ *quasiuniformly converges to* $f : X \to Y$, if it pointwise converges to f and for every $\epsilon > 0$ and for every natural number m, there exists a natural number p such that for all $x \in X$:

$$\min \{d(f_{m+1}(x), f(x)), \ldots, d(f_{m+p}(x), f(x))\} < \epsilon.$$

If $\mathcal{F} \subset Y^X$, we denote by $U(\mathcal{F})$, $D(\mathcal{F})$, and $P(\mathcal{F})$ the collection of all uniform, quasiuniform, and pointwise limits of sequences taken from $\mathcal{F}$, respectively.

A function $f : X \to Y$ is *quasicontinuous* if $f^{-1}(V) \subset \mathrm{Cl\ Int}\ f^{-1}(V)$ for each open set V in Y; f is *simply continuous* if $f^{-1}(V)$ is a union of an open set and a nowhere dense set for each open set V in Y; f is *cliquish* if for each $x \in X$, for each $\epsilon > 0$ and each neighborhood U of x there is an open nonempty set $G \subset U$ such that $d(f(y), f(z)) < \epsilon$ for each $y, z \in G$.

Denote by Q, S, K, and B the set of all functions which are quasicontinuous, simply continuous, cliquish, and having the Baire property (with X as the domain and Y as the range), respectively. Evidently, $Q \subset K \subset B$ and $Q \subset S \subset B$. In [5] it is shown that if X is a Baire space and Y is a separable metric space, then $S \subset K$.

It is well–known that $U(Q) = Q$, $U(K) = K$ and $P(B) = B$. In [1] it is shown that $D(K) = K$ for a Baire space X. In [2] (also [4]) it is shown that for $X = \mathbb{R}^m$ and $Y = \mathbb{R}$ we have $P(Q) = K$ and $P(K) = B$.

Theorem 1 *Let X be a Baire space and (Y, d) be a separable locally compact metric space. Then $P(S) = B$ and $U(S) = K$.*

Theorem 2 *If $X = Y = \mathbb{R}$, then $D(Q) = K$.*

Therefore, for real functions of a real variable we obtain the following complete description (where the inclusions are proper):

Theorem 3 *Let* $X = Y = \mathbb{R}$. *Then* $Q = U(Q) \subset S \subset K = U(K) = D(K) = U(S) = D(S) = D(Q) = P(Q) \subset B = U(B) = D(B) = P(B) = P(K) = P(S)$.

By [3] every cliquish $f : \mathbb{R} \to \mathbb{R}$ is a sum of 4 quasicontinuous functions and by [6] (also [4]) every cliquish function $f : \mathbb{R}^m \to \mathbb{R}$ is a sum of 6 quasicontinuous functions.

Theorem 4 *Every cliquish function* $f : \mathbb{R}^m \to \mathbb{R}$ *is a sum of 4 quasicontinuous functions and it is a sum of 2 simply continuous functions.*

References

[1] J. Doboš and T. Šalát, Cliquish functions, Riemann integrable functions and quasiuniform convergence, *Acta Math. Univ. Comen.* 40-41 (1982), 219–223.

[2] Z. Grande, Sur la quasi-continuité et la quasi–continuité approximative, *Fund. Math.* 129 (1988), 167–172.

[3] Z. Grande, Sur les fonctions cliquish, Časopis pěst. mat. 110 (1985), 225–236.

[4] Z. Grande, T. Natkaniec, and E. Stronska, Lattices, algebras and Baire's systems generated by some families of functions, *Real Analysis Exch.* 13 (1987–88), 61–66.

[5] A. Neubrunnová, On certain generalizations of the notion of continuity, *Mat. časopis* 23 (1973), 374–380.

[6] E. Stronska, L'espace linéaire des fonctions cliquées sur $\mathbb{R}^n$ est généré par les fonctions quasi–continuous, *Math. Slovaca* 39 (1989), 155–164.