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Typical Functions in Subspaces of $\mathcal{B}^1$: Thick or Thin ?

We present the main results from recent work [2], [3] on problems stated in [1]. These problems concern typical behaviour in subspaces of $b\mathcal{B}^1$; i.e. the Zahorski classes, the space of all (bounded) derivatives and the space of all continuous functions, all on $[0,1]$. The emphasis is put on those properties which distinguish these spaces, i.e. they are in some but not in all of them. The most instructive result is the following: A typical function (i.e. any function except a first category subset of the function space under consideration) in $b\mathcal{DB}^1([0,1])$ as well as in $\mathcal{C}([0,1])$ has all level sets of Hausdorff dimension zero and a graph of Hausdorff dimension one (i.e. is very "thin"), but a typical approximately continuous function has all (except the top and bottom) level sets of Hausdorff dimension one and a graph of Hausdorff dimension two (i.e. is very "thick"). This demonstrates the situation when a property is typical in the basic space, a "very opposite" property is typical in a subspace, but in a further subspace of this subspace the original property is typical again. Since there is no linkage between the definitions of the property and of the subspaces, this example advises to be careful in using "interpolation principles".

References

[1] A. M. Bruckner and G. Petruska, Some Typical Results on Bounded Baire 1 Functions, *Acta Math. Hung.*,**43**(1984), 325–333.

[2] B. Kirchheim, Some Further Typical Results on Bounded Baire One Functions, *to appear in Acta Math. Hung.*, **62**(1993), no. 3–4.

[3] B. Kirchheim, Typical approximately continuous functions are surprisingly thick, *submitted to Real Analysis Exchange.*