Ľ. Snoha, Department of Mathematics, Faculty of Education, Tajovského 40, 975 49 Banská Bystrica, Czechoslovakia
S. F. Kolyada, Institute of Mathematics, Ukrainian Academy of Sciences, Repin str. 3, 252601 Kiev–4, USSR

On ω–Limit Sets of Triangular Maps

As established in [1] and [2], a nonvoid closed subset M of $I = [0, 1]$ is an ω–limit set for some continuous function $f : I \mapsto I$ if and only if M is nowhere dense or i s a union of finitely many nondegenerate closed intervals.

A continuous map $F : I^2 \mapsto I^2$ is called a *triangular* map if $F(x, y) = (f(x), g(x, y))$, i.e. if the first coordinate of the image of a point depends only on the first coordinate of that point. The triangular map F splits the square I^2 into one–dimensional fibres (intervals x = constant) such that each fibre is mapped by F into a fibre. Denote by $C_\Delta(I^2, I^2)$ the set of all continuous triangular maps from I^2 into itself and by $\omega_f([x, y])$ the ω–limit set of the point $[x, y]$ under F.

Our main result is the characterization of those ω–limit sets of triangular maps which lie in one fibre. Trivially, as an ω–limit set lying in a fibre $I_a = \{a\} \times I$ we can get any set of the form $\{a\} \times M$ where M is a set which can serve as an ω–limit set for a continuous map from I into I. But it turns out that many other sets can also be obtained. The complete answer is given by

Theorem 1 *For $a \in I$, $M \subset I$ the following two conditions are equivalent:*

1. *There is $F \in C_\Delta(I^2, I^2)$ and a point $[x, y] \in I^2$ with $\omega_F([x, y]) = \{a\} \times M$;*

2. *M is a nonempty closed subset of I which is not of the form*

$$M = J_1 \cup J_2 \cup \ldots J_n \cup C, \tag{1}$$

where n is a positive integer, the J_i, $i = 1, 2, \ldots, n$, are closed intervals, C is a nonempty countable set,and all the sets J_i and C are mutually disjoint and dist$(C, J_i) > 0$ for at least one $i \in \{1, 2, \ldots, n\}$.

Using Theorem 1 it is easy to show that if A is a nonempty finite set, then $A \times M$ is an ω–limit set for a continuous triangular map if and only if M is a nonempty closed subset of I which is not of the form (1).

Now suppose that an ω–limit set is not a subset of a fibre. Then the question is whether any closed subset of a fibre can be obtained as an intersection of this fibre and an ω–limit set of an $F \in C_\Delta(I^2, I^2)$. The answer is affirmative.

Theorem 2 *Let $a \in I$, and let M be any closed subset of I. Then there are $F \in C_\Delta(I^2, I^2)$ and $[x, y] \in I^2$ with $\omega_f([x, y]) \cap I_a = \{a\} \times M$.*

References

[1] S. J. Agronsky, A. M. Bruckner, J. G. Ceder, and T. L. Pearson, The structure of ω–limit sets, *Real Analysis Exch.* 15 (1989-90), 483–510.

[2] A. M. Bruckner and J. Smítal, The structure of ω–limit sets for continuous maps of the interval, *Math. Bohemica* (to appear).