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On Approximation Spaces

The notion of approximation space connected with computer data analysis has been defined by Pawlak [3] as a pair (U, R), where U is a non-empty set and R is an equivalence relation. Any equivalence relation on U corresponds in a unique way to a certain partition P of U, and this gives rise to an approximation space (U, P). The concept of measurable rough structure has been introduced by Iwiński [1], and in his paper Iwiński mentions that there are some apparent similarities between rough measures and the interval representations of some types of indiscernibility structures considered in measurement theory, see for example [3].

The aim of this paper is to express the notions of upper and lower approximations in terms of multifunctions. Let T and U be non-empty sets, $F : T \to U$ a strict surjective multifunction and $\mathcal{F}(T) = \{F(t) : t \in T\}$. Then $(U, \mathcal{F}(T))$ is an approximation space. If $X \subset U$, then the lower approximation of X is given by $F(F^+(X))$ and the upper approximation of X is given by $F(F^-(X))$, where F^+ and F^- are the strong and the weak inverses of F, respectively.

If F is image-non-mingled, then $\mathcal{F}(T)$ is a partition of U, and FF^- is a closure operation while FF^+ is an interior operation on the power set of U. The topologies induced by these operations on U are identical. Let $G(x) = F(F^-(\{x\}))$ for every $x \in U$, and let $G_1(X) = G^+(X) \cap X$ for every $X \subset U$; G_1 is the weak lower approximation of X. If $\mathcal{F}(T)$ is a covering of U, then the following are equivalent : (1) $\{G(x) : x \in U\}$ is a partition of U; (2) FF^- is a closure operation; (3) G_1 is an interior operation. Let G_2 be defined by $G_2(X) = U \backslash (F(F^+(U \backslash X)))$ for every $X \subset U$; G_2 is the strong upper approximation of X. The operation FF^+ is an interior operation if and only if the covering $\mathcal{F}(T)$ is a base for a topology on U. In this case, FF^+ induces the conjugated closure operation G_2. Several more topological properties of FF^+, FF^-, G_1 and G_2 are derived, with the aim of applications in mathematical economics and in the theory of integration of multifunctions.

References

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