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A Parametrization Theorem for Generalized Borel Sets

Let $(X, \mathcal{A})$ be an arbitrary **Borel structure** (= measurable space); let $(\mathbb{R}, \mathcal{B})$ denote the real line with usual Borel structure, and let $(X \times \mathbb{R}, \mathcal{A} \times \mathcal{B})$ denote the product Borel structure. Let E be the ordinate set $\{(x,t) : x \in X, 0 \leq t \leq \Phi(x)\}$ of an $(\mathcal{A}, \mathcal{B})$ measurable positive function Φ, and let $G_1, G_2, \ldots$ be the graphs of $(\mathcal{A}, \mathcal{B})$ measurable functions $g_n : A_n \mapsto \mathbb{R}$ (where each $A_n \in \mathcal{A}$). It is known that there is an $(\mathcal{A}, \mathcal{B})$ isomorphism, preserving first coordinates and sectional Lebesgue measure, of $E \setminus \cup_n G_n$ onto E. This generalizes a special case of a theorem of R. D. Mauldin ([2]; cf. [1], [3]) — special in that the deleted sectionally-null set is countable, general in that $(X, \mathcal{A})$ is quite arbitrary.

References

[1] D. Maharam, On the planar representation of a measurable subfield, *Lecture Notes in Math.* 1089, Springer Verlag 1984, 47–57.

[2] R. D. Mauldin, Borel parametrizations, *Trans. Amer. Math. Soc.* 250 (1979), 223–234.

[3] R. D. Mauldin, D. Preiss, and H. von Weiszacker, Orthogonal transition kernels, *Ann. of Probability* 11 (1983), 970–988.