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Some Inversion Results for the Generalized Walsh–Fourier Transform

In the trigonometrical case it is known (see [3]) that if the integral

$$\int_{-\infty}^{\infty} e^{ixy}\alpha(x)dx,$$

where $\alpha(x)$ is locally summable, converges everywhere to a function $f(y)$ which is finite and locally summable, then

$$\alpha(x) = (C,1)\frac{1}{2\pi}\int_{-\infty}^{\infty} f(y)e^{-ixy}dy \text{ for almost all } x.$$

We consider here a similar problem for the multiplicative transform. One of the results in this direction is the following

Theorem 1 *Let the integral*

$$\int_{0}^{\infty} \alpha(x)\chi(x,y)dx,$$

where $\chi(x,y)$ is a continual analogue of the Vilenkin multiplicative system (see [1]) and $\alpha(x)$ is locally summable on $[0,\infty)$, converges everywhere to a function $f(y)$ which is finite and locally summable on $[0,\infty)$. Then

$$\alpha(x) = \lim_{n\to\infty}\int_{0}^{m_n} f(y)\overline{\chi(x,y)}dy \ \textit{a.e. on } [0,\infty),$$

where $\{m_n\}$ is an increasing sequence of positive integers, uniquely defined by the kernel $\chi(x,y)$.

Lebesgue integration in this statement can be replaced by Perron integration.

A special case of the above theorem with $f(x) = 0$ was proved by Vilenkin [2].

References

[1] B. Golubov, A. Efimov, and V. Skvortsov, *Walsh Series and Transforms*, Kluwer Academic Publisher, 1991.

[2] N. Vilenkin, On the theory of Fourier integral on topological groups, *Mat. Sbornik* 30 (1952), 233–244.

[3] A. Zygmund, *Trigonometric Series*, Cambridge University Press, New York, Vols. I, II, 1959.