L. Zajíček, Department of Mathematical Analysis (MFF UK), Charles University, Sokolovská 83, 186 00 Prague, Czechoslovakia

A Note on σ–Porous Sets

It is well–known (cf. [3]) that there exists a measure zero closed nowhere dense set $F \subset \mathbb{R}^n$ which is not σ–porous. The following theorem is a generalization and improvement of this fact.

Theorem 1 *Let X be a topologically complete metric metric space having no isolated points. Then there exists $F \subset X$ such that*

1. *F is closed and nowhere dense,*
2. *F is not σ–porous, and*
3. *if X is separable, then the Hausdorff dimension of F is zero.*

The (relatively complicated in terms of details) construction of F is based on a generalized Foran's lemma [3].

The following theorem was proved by my student P. Šleich [2]. (There is an unpublished manuscript in English containing the proof.)

Theorem 2 *Each set $A \subset \mathbb{R}$ of type $H^{(s)}$ $(s = 1, 2, \ldots)$ is σ–porous.*

Note that each set of type $H^{(s)}$ is a U-set in the sense of the theory of trigonometric series (cf. [1]). It is not known whether each closed U-set is σ–porous (cf. [3], p. 345).

References

[1] N. K. Bari, *Trigonometrical Series*, Moscow, 1961.

[2] P. Šleich, Thesis, Charles University, Prague, 1991.

[3] L. Zajíček, Porosity and σ–porosity, *Real Analysis Exch.* 13 (1987–88), 314–350.