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Bi-Lipschitz Mappings and Density Points

W.F.Pfeffer asked the author about how density points of Lebesgue measurable sets are transformed by bi-Lipschitz mappings. We call a mapping f bi-Lipschitz if f and f^{-1} are both Lipschitz. By some authors bi-Lipschitz mappings are called lipeomorphisms. In [B1] we prove that density and dispersion points are preserved by bi-Lipschitz mappings. The "almost every" version of our theorem, that is, almost every density points are mapped into density points is easy. The point in our theorem is that we prove our result about all density points.

Theorem 1 in [B1] implies that each bi-Lipschitz function maps the essential boundary of its domain onto the essential boundary of its image. This corollary of Theorem 1 motivated W.F.Pfeffer's original question. In conjunction with [F, Theorem 4.5.11] the above corollary implies that the bi-Lipschitz image of a *Caccioppoli set* is again a Caccioppoli set. The usual proof of this fact rests on interpreting Caccioppoli sets as *integral currents* - a technique substantially more involved than the one presented in [B1]. (cf. [F, Chapter 4]).

We found interesting that the topic discussed in [B1] is related to Topology, namely, to Brouwer's Theorem on the Invariance of Domain. In [B2] we give a discrete version of this theorem.

References

[B1] Z. Buczolich, Density Points and Bi-Lipschitz Functions in \mathbb{R}^m , Proc. Amer. Math. Soc., to appear.

[B2] Z. Buczolich, A Discrete Version of Brouwer's Theorem on the Invariance of Domain, submitted.

[F] H. Federer, Geometric Measure Theory, Springer-Verlag, 1969.