THE DIFFERENTIABILITY STRUCTURE OF TYPICAL FUNCTIONS IN C [0,1]

L.Zajíček, Prague

1. Introduction. Let C denote the set of continuous real valued functions defined on [0,1] furnished with the metric of uniform convergence. When we say a typical $f \in C$ has a certain property P , we shall mean that the set of $f \in C$ with this property is residual in C . The sentences " a.a. (= almost all) $f \in C$ have the property P " and " P is a typical property of $f \in C$ " have the same meaning.

The following theorem show relations connecting the Dini derivates of a.a. $f \in C$ at all points $x \in (0,1)$.

<u>Theorem BMJ.</u> A typical $f \in C$ has the following properties.

(i) ([1],[6]) At every $x \in (0,1)$, $\max(|D^+f(x)|, |D_+f(x)|) = \infty$ and $\max(|D^-f(x)|, |D_-f(x)|) = \infty$.

(ii) ([3]) At every $x \in (0,1)$, $[D_-f(x), D^-f(x)] \cup [D_+f(x), D^+f(x)] = [-\infty, \infty]$.

A natural problem (cf. [2] ,Remark 2) arises, wheather there are some further relations of this sort.

The well-known Saks' result [8] says that a.a. $f \in C$ have a right-sided derivative ∞ in a non-denumerable set of points.

Garg ([2], Theorem 1, (iii)) observed that, for any $r \in \mathbb{R}$, the case $D^+f(x) = \infty$, $D_-f(x) = -\infty$, $D_+f(x) = D^-f(x) = r$ occurs for a.a. $f \in C$ on a dense set.

The problem mentioned above was solved in negative approximately six years ago by D.Preiss.

<u>Theorem P .</u> (Preiss, unpublished.) Let D^+, D_+ , D^-, D_- be extended real numbers for which $\max(|D^+|, |D_+|) = \max(|D^-|, |D_-|) = \infty$ and $[D_-, D^-] \cup [D_+, D^+] = [-\infty, \infty]$. Then for a.a. $f \in C$ there exists a $\mathcal{C}$-dense set $A \subset (0,1)$ such that

$D^+f(x) = D^+$, $D_+f(x) = D_+$, $D^-f(x) = D^-$, $D_-f(x) = D_-$

for all $x \in A$.

An improvement of Theorem P (Theorem 2) is given below.

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Institute of Mathematics
Łódź University
ul. Stefana Banacha 22
90-238 Łódź, POLAND