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TYPICAL CONTINUOUS FUNCTIONS ARE NOT ITERATES

Let C_O^1 denote the set of continuous functions mapping $[O,1]$ into itself endowed with the sup norm. P. D. Humke and M. Laczkovich have investigated the structure of the set $W_2 = \{f \circ f : f \in C_O^1\}$. They proved that W_2 is not everywhere dense in C_O^1 and does not contain balls.

In this presentation we shall see that W_2 is of the first category in C_O^1. Moreover, for $k \geq 2$, if W_k denotes the set of k^{th} iterates of the functions in C_O^1, then W_k is shown to be of first category in C_O^1 as well. The proof is based on the theorem stated below. First some notation needs to be adopted.

Let $\mathrm{Fix}(f) = \{x : f(x) = x\}$ and let $f^n(x) = f(f^{n-1}(x))$. We set $E_\alpha^f = \{x : f(x) = \alpha\}$ and let RE_f denote the set of relative extrema of f. We define

$T_1 = \{f \in C_O^1 : \mathrm{Fix}(f) \cap RE_f = \emptyset\}$,

$T_2 = \{f \in C_O^1 : f$ is not locally increasing or decreasing at any $x \in [O,1]\}$,

$T_3 = \{f \in C_O^1 : \mathrm{Card}(E_\alpha^f \cap RE_f) \leq 1$ for every $\alpha\}$, and

$T = T_1 \cap T_2\, T_3$.

It is easy to see that T is a residual subset in C_O^1. We then prove the following:

Theorem. For every $\varphi \in C_O^1$ and $k \geq 2$, $\varphi^k \notin T$.