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When does $\lim f(\lambda_n x) = 0$ for every x imply $f(x) \to 0$ at infinity?

The following problem is due to M. Laczkovich and L. Pósa. We have a continuous function f and we know that for every $x > 0$ $f(nx)$ tends to zero as n tends to infinity. Prove that $\lim_{x \to \infty} f(x) = 0$.

This observation leads to the following question. Which sequences have this property besides the sequence of all natural numbers?

<u>Definition</u>: A real sequence $\{\lambda_n\}$ is <u>good</u> if $\lambda_n \to \infty$ and, for any continuous f, $\lim_{n \to \infty} f(\lambda_n x) = 0$.for all $x > 0$ implies $\lim_{x \to \infty} f(x) = 0$.

In this paper we give a sufficient and some necessary conditions. First of all we give an equivalent reformulation of the problem.

<u>Proposition</u>: A sequence $\{\lambda_n\}$ is good iff $\lambda_n \to \infty$ and whenever $\{[a_i, b_i]\}$ is a sequence of intervals tending to infinity then there exists an x such that $\lambda_n x \in \bigcup_{i=1}^{\infty} [a_i, b_i]$ for infinitely many n.

The solution of the original problem is based on the fact that $\bigcup_{n=1}^{\infty} [nx, ny]$ contains a halfline for an arbitrary interval x,y . Similar argument shows that if $\lambda_{n+1}/\lambda_n \to 1$ then $\{\lambda_n\}$ is good.

With the help of the fact that if $\limsup \lambda_{n+1}/\lambda_n = \infty$ then $\bigcup_{n=1}^{\infty} [\lambda_n x, \lambda_n y]$ does not contain a halfline for any interval $[x, y]$ one can prove that.if $\limsup \lambda_{n+1}/\lambda_n = \infty$ then $\{\lambda_n\}$ is not good. This observation is due to P. Erdős.

A little more complicated argument shows that if $\lim \lambda_{n+1}/\lambda_n = c > 1$ then $\{\lambda_n\}$ is not good. We have found some sequences which are not good but do not satisfy any of the last two conditions.

A stronger property for $\{\lambda_n\}$ can be completely characterized.

<u>Definition</u>: A sequence $\{\lambda_n\}$ is <u>very good</u> if $\lambda_n \to \infty$ and whenever f is a continuous function and [a,b] is an arbitrary interval such that $\lim_{n\to\infty} f(\lambda_n x) = 0$ for every $x \in [a,b]$, then $\lim_{x\to\infty} f(x) = 0$.

<u>Proposition:</u> is very good iff and lim = 1.

Further results concerning the characterization of good sequences were obtained recently by P. Erdős, M. Laczkovich and the author.