*Real Analysis Exchange Vol. 4 (1978-79)*

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## Concerning Query 37

Call a subset of the real line R symmetric if for each $x \in R$, there is a positive $\delta_x$ such that for all $0 < h < \delta_x$, $x+h \in E$ if and only if $x-h \in E$. Must such a set E be Lebesgue measurable?

This question appeared as Query 37 in Volume 3 Number 2 of the Exchange. The answer to the question is yes and it is well documented in the literature. A function f is said to have a zero symmetric derivative at x if $f(x+h) - f(x-h) = o(h)$ as $h \to 0$. Z. Charzynski [1] proved that if f has a zero symmetric derivative at each $x \in R$, then there is a constant c such that $f(x)=c$ except for those x in a nowhere dense countable set. Consequently, if E is a symmetric set in the sense of the query, either it or its complement must be a nowhere dense countable set.

I. Z. Ruzsa has provided an elementary proof of the sharper result that either E or its complement must have nowhere dense countable closure. This result is presented next in this inroads section of the Exchange.

Roy O. Davies has indicated an alternate method of obtaining this sharper result by incorporating a known result concerning symmetrically continuous functions. His note appears immediately following Ruzsa's.

## Reference

[1] Z. Charzyński, Sur les fonctions dont la dérivée symetrique est partout finie, Fundam. Math 21(1933), 214-225.