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CORRECTIONS TO MY PAPER "ZETA FUNCTIONS OF INTEGRAL GROUP RINGS OF METACYCLIC GROUPS"

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In that paper, the countings of the local isomorphism classes were incorrect. For the case $\Lambda=\boldsymbol{Z}_pG_p$, $G_p=C_p\cdot C_q$, p and q are primes and C_q acts faithfully on C_p (cf. §3), there are $\sum_{t=1}^{q}\binom{q}{t}2^t\cdot t^{q-t}$ isomorphism classes of full Λ-lattices in $\Lambda\boldsymbol{Q}_p$, where $\binom{q}{t}$ is the binomial coefficient. In fact, let $\mathfrak{M}=\{\boldsymbol{Z}_p[\varepsilon_p]\circ C_q$-lattices of $\boldsymbol{Z}_p[\varepsilon_p]$-rank $q\}$. Then every full Λ-lattice M in $\boldsymbol{Q}_p\Lambda$ can be realized as an extension of $\boldsymbol{Z}_pC_q$ by some M_0 in $\mathfrak{M}$, and from non-isomorphic M_0's, we have non-isomorphic Λ-lattices. Let $P=(\varepsilon_p-1)\boldsymbol{Z}_p[\varepsilon_p]$ and $P^0=\boldsymbol{Z}_p[\varepsilon_p]$. For a subset T of $\{0, 1, \cdots, q-1\}$ such that $\sharp(T)=t\geqq 1$, define $\mathfrak{M}_T$ by

$$\mathfrak{M}_T=\{M_0\in\mathfrak{M}\mid M\oplus\!\!> \bigoplus_{j\in T}P^j,\ i\in T \text{ if } M\oplus\!\!> P^i\}.$$

Then $\sharp(\mathfrak{M}_T)=t^{q-t}$ and we see that $\dot{\bigcup}_{T\subseteqq\{0,1,\cdots,q-1\}}\mathfrak{M}_T$ (disjoint union) is a full set of representatives of the isomorphism classes of $\mathfrak{M}$. There are 2^t isomorphism classes in $\mathrm{Ext}_\Lambda(\boldsymbol{Z}_pC_q, M_o)$ for each $M_0\in\mathfrak{M}_T$. If $t\,(1\leqq t\leqq q)$ is given, there are $\binom{q}{t}$ choices of subsets T of $\{1, 2, \cdots, q-1\}$ such that $\sharp(T)=t$. Thus we have the required number.

In the case $q=2$, i.e. G_p is the dihedral group D_p of order $2p$, we shall correct Proposition 3.6. and Theorem 3.7. to the form:

$$\zeta(\boldsymbol{Z}_pD_p;\ s)=\frac{1+(p-1)t^2+2p^2t^3+(p^2-p)t^4+p^3t^6}{(1-t)^2(1-t^2)(1-pt^2)},\quad \text{where}\quad t=p^{-s};$$

$$\begin{aligned}\zeta(\boldsymbol{Z}D_p;\ s)=&\zeta_{\boldsymbol{Z}}(s)^2\zeta_R(2s)\zeta_R(2s-1)\\&\times(1-2^{-s}+2^{1-2s})(1+(p-1)t^2+2p^2t^3+(p^2-p)t^4+p^3t^6),\end{aligned}$$

where $R=\boldsymbol{Z}[\varepsilon_p+\varepsilon_p^{-1}]$ and $t=p^{-s}$.

For the case $\Lambda=\boldsymbol{Z}_p[\xi_d]\circ G_p$ (cf. §4), there are $q+1$ isomorphism classes of full Λ-lattices in $\boldsymbol{Q}_p$, and $L_{(\underbrace{1,\cdots,1}_{r},0,\cdots,0)}$, $0\leqq r\leqq q$, are the representatives. Hence, we shall correct Proposition 4.5 to the form:

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$$\zeta(\boldsymbol{Z}_p[\xi_d]\circ G_p\,;\, s)=\sum_{r=0}^{q}\sum_{h=0}^{r}\left(\prod_{i=h}^{r-1}\frac{(P^q-P^i)^2}{P^r-P^i}\times\prod_{i=0}^{h-1}\frac{P^q-P^i}{P^h-P^i}\times\frac{P^{-q(q+r-2h)s}}{\prod_{i=h}^{q-1}(1-P^{i-qs})^2}\right).$$

Finally, we have, in stead of Theorem 4.6,

$$\zeta(\boldsymbol{Z}D_n\,;\, s)=\zeta_{\boldsymbol{Z}}(s)^2\Big(\prod_{\substack{d\mid n\\ d\neq 1}}\zeta_{R_d}(2s)\zeta_{R_d}(2s-1)\Big)$$
$$\times(1-2^{-s}+2^{1-2s})\prod_{p\mid n}\Big(F_{p,1}(s)\prod_{\substack{d\mid n/p\\ d\neq 1}}F_{p,d}(s)^{g_d}\Big),$$
$$F_{p,1}(s)=1+(p-1)t^2+2p^2t^3+(p^2-p)t^4+p^3t^6 \qquad (t=p^{-s}),$$

and for $d\neq 1$,

$$F_{p,d}(s)=1-(P_d+1)T_d^2+(P_d^3+2P_d^2+3P_d)T_d^4-(P_d^3+P_d^2)T_d^6+P_dT_d^8 \quad (T_d=P_d^{-s}),$$

where for each $p\mid n$ and $d\mid n/p$, $d\neq 1$, g_d is the number of distinct prime ideals over (p) in $R_d=\boldsymbol{Z}[\varepsilon_d+\varepsilon_d^{-1}]$ and $P_d=p^{\phi(d)/2g_d}$.

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References

[1] Hironaka, Y., Zeta functions of integral group rings of metacyclic groups, Tsukuba J. Math. 5(2) (1981), 267-283.

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