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COMMUTATORS OF TWO COMPRESSED SHIFTS AND THE HARDY SPACE ON THE BIDISC

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Dedicated to Professor Tsuyoshi Ando on his eightieth birthday

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ABSTRACT. For a subset E of the bidisc $D^2, M = \{f \in H^2(D^2) \ : \ f = 0$ on $E\}$ and N is the orthogonal complement of M in $H^2(D^2)$ where $H^2(D^2)$ is the two variable Hardy space on D^2. We describe the finite rank commutants of the restricted shifts S_z and S_w on N when E satisfies some natural condition. Moreover we give a sufficient condition for that the Pick interpolation is possible.

1. INTRODUCTION

For $1 \leq p \leq \infty$, $H^p(D)$ denotes the one variable Hardy space on the open unit disc D in $\mathbb{C}$ and $H^p(D^2)$ denotes the two variable Hardy space on $D^2 = D \times D$. Let m be the normalized Lebesgue measure on $T^2 = T \times T$ when $T = \partial D$. Each f in $H^2(D^2)$ has a radial limit f^* defined on T^2 $a.e.m$ and let $H^p(T^2) = \{f^* : f \in H^p(D^2)\}$. Then $H^p(T^2)$ is a Banach space in $L^p(T^2) = L^p(T^2, m)$. It is known that $H^p(D^2)$ is isometrically isomorphic to $H^p(T^2)$. These facts are shown in [5]. z and w are the coordinates of the functions on $\mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$

A closed subspace M of $H^2(D^2)$ is said to be an invariant subspace if $zM \subset M$ and $wM \subset M$. Let N be the orthogonal complement of M in $H^2(D^2)$. In this paper, we assume that

$$M = \{f \in H^2(D^2) : f = 0 \text{ on } E\}$$

for a subset E of D^2. Then N is the closed linear span of $\{(1 - \bar{a}z)^{-1}(1 - \bar{b}w)^{-1} : (a, b) \in E\}$. Hence $N \cap H^\infty(D^2)$ is dense in N.

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For a function ψ in $H^2(D^2)$, put

$$S_\psi f = P^N(\psi f) \quad (f \in N \cap H^\infty(D^2))$$

where P^N is the orthogonal projection onto N. We do not know whether S_ψ is bounded or not. The following question is natural. Does there exist a function ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi$ when S_ψ is bounded? However this question is answered negatively [2]. In this paper we consider the following problem.

Problem 1.1. Let ψ be a function in $H^2(D^2)$. If S_ψ is of finite rank then does there exist a function ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi$?

If A is a bounded linear operator on N such that $AS_z = S_z A$ and $AS_w = S_w A$ then it easy to see that $A = S_\psi$ on $N \cap H^\infty(D^2)$ for some ψ in $H^2(D^2)$. Conversely if $A = S_\psi$ on $N \cap H^\infty(D^2)$ for some ψ in $H^2(D^2)$ then $AS_z = S_z A$ and $AS_w = S_w A$ on $N \cap H^\infty(D^2)$. For one variable $H^2(D)$, this is in [6]. The same proof is valid in two variable $H^2(D^2)$. Now it follows that $AS_z = S_z A$ and $AS_w = S_w A$ on N because $N \cap H^\infty(D^2)$ is dense in N. Hence if the problem above can be solved positively then finite rank operators of commutants of S_z and S_w are described.

For an arbitrary invariant subspace M, Sarason [6] solved this problem in 1967 for one variable $H^\infty(D)$ without the finite rank condition. Hence if ψ is one variable then the above problem can be solved.

Suppose that $\{\zeta_j\}_{j=1}^n = \{(a_j, b_j)\}_{j=1}^n$ is in D^2 and $\{\eta_j\}_{j=1}^n$ is in $\mathbb{C}$. When $\zeta = (z, w)$, $k_{\zeta_j}(\zeta)$ denotes the reproducing kernel of ζ_j for $H^2(D^2)$, that is, $k_{\zeta_j}(\zeta) = (1 - \bar{a}_j z)^{-1}(1 - \bar{b}_j w)^{-1}$. In this paper we consider the following problem.

Problem 1.2. If the Pick matrix

$$[(1 - \eta_i \bar{\eta}_j) k_{\zeta_j}(\zeta_i)] \geq 0 \quad (1 \leq j \leq n)$$

then does there exist a function ϕ in $H^\infty(D^2)$ such that $\|\phi\|_\infty \leq 1$ and $\phi(\zeta_j) = \eta_j$ $(1 \leq j \leq n)$?

In Problem 1.2, it is easy to see the converse is valid. Unfortunately it is known [1] that Problem 1.2 can be solved negatively in general. Hence we will consider it in some special case.

G. Pick solved this problem in 1916 for one variable $H^\infty(D)$. Hence if $\zeta_j = (a_j, b_j)$ $(1 \leq j \leq n)$ and $b_1 = b_2 = \cdots = b_n$ then the above problem can be solved. It should be noted that Agler and M^{c}Carthy [1] solved a two variable interpolation problem in a different form from the above problem.

For a subset E of D^2, put $E_{a\cdot} = \{b \in D : (a, b) \in E\}$ and $E_{\cdot b} = \{a \in D : (a, b) \in E\}$.

2. Problem 1

In this section we solve Problem 1.1 under some condition. We do not use Sarason's generalized interpolation theorem [6]. For $0 < p \leq \infty$, if $E \subset D^2$ and $c \in E$ then put

$$\rho_p(c) = \rho_p(c, E) = \sup\{|f(c)| : f \in H^p(D^2), \|f\|_p \leq 1 \text{ and } f = 0 \text{ on } E \backslash \{c\}\}.$$

In general, it may not be true that $\rho_p(a) = \rho_q(a)$ for $p \neq q$. If E is a finite set, then $\rho_p(c) > 0$ for $0 < p \leq \infty$. But in general it may happen that $\rho_p(c) = 0$ for $0 < p \leq \infty$. In fact, suppose $E = \{(0, w) : w \in D\}$ then $\rho_p(c) = 0$ for any $c \in E$. There exists a set E in D^2 such that $\rho_\infty(c) = 0$ and $\rho_2(c) > 0$. In fact, we can find such a set in $\{(a_j, a_j) \in D^2 : \sum_{j=1}^\infty (1 - |a_j|^2) = \infty\}$. For $\{f(z, z) : f \in H^2(D^2)\}$ is a one variable Bergman space L_a^2 (see [5, p.53]) and $\{f(z, z) : f \in H^\infty(D^2)\}$ is just $H^\infty(D)$. It is known that there is a nonzero function in $L_a^2(D)$ whose zero set does not satisfy a Blaschke condition.

Theorem 2.1. *Suppose $M = \{f \in H^2(D^2) : f = 0$ on $E\}$ for a subset E of D^2 and $\rho_\infty(a) > 0$ when $\rho_2(a) > 0$ for a in E. If ψ is a function in $H^2(D^2)$ such that S_ψ is of finite rank on N the orthogonal complement of M, then there exists a function ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi$.*

Proof. Suppose S_ψ is of finite rank n. If $n = 0$ then the theorem is clear and so we may assume $n > 0$. Then the range of S_ψ is of n dimension and so there exist k_j in N $(1 \leq j \leq n)$ such that $S_\psi N =$ the linear span $\langle k_1, \cdots, k_n\rangle$ of $k_1, \cdots, k_n$. Since S_z and S_w commute with S_ψ (see Introduction),$\langle k_1, \cdots, k_n\rangle$ is an invariant subspace of S_z and S_w. Hence there exist one variable minimal polynomials $p = p(z)$ and $q = q(w)$ such that $p(S_z)k_j = S_p k_j = 0$ and $q(S_w)k_j = S_q k_j = 0$ for $1 \leq j \leq n$. Therefore pk_j and qk_j belong to M for $1 \leq j \leq n$. Then we may assume zeros of p and q are all simple by the definition of M. Hence for all f in N $pS_\psi f = qS_\psi f = 0$ and so $p\psi f = q\psi f = 0$ on E. For each $(x, y) \in E$, there is a function F in H^2 with $F(x, y) \neq 0$. Put $f = P^N F$ then $f(x, y) = F(x, y)$ because $(I - P^N)F = 0$ on E. Therefore $p(x)\psi(x, y) = q(y)\psi(x, y) = 0$ for any $(x, y) \in E$. Thus

$$\psi = 0 \text{ on } E\backslash\{(x, y) \in E : p(x) = q(y) = 0\}.$$

Hence there exist $(a_1, b_1), \cdots, (a_m, b_m)$ in E such that $\psi = 0$ on $E\backslash \bigcup_{j=1}^m (a_j, b_j)$. Since p and q are minimal polynomials of S_z and S_w, respectively, $\psi(a_j, b_j) \neq 0$ if $p(a_j) = q(b_j) = 0$. For each $1 \leq j \leq m$, there exists a $f_j \in H^\infty(D^2)$ such that $f_j(a_j, b_j) = 1$ and $f_j(a_\ell, b_\ell) = 0$ if $j \neq \ell$. Put $\psi_j = f_j\psi$ then $\psi_j(a_j, b_j) = \psi(a_j, b_j) \neq 0$ and so $\rho_2((a_j, b_j)) > 0$. Since $\rho_\infty((a_j, b_j)) > 0$ for $1 \leq j \leq m$ by the hypothesis, there exist h_j in $H^\infty(D^2)$ such that

$$h_j(a_j, b_j) \neq 0 \text{ and } h_j = 0 \text{ on } E\backslash(a_j, b_j).$$

Hence there exist $\alpha_1, \cdots, \alpha_n$ in $\mathbb{C}$ such that $\psi - \sum_{j=1}^m \alpha_j h_j \in M$. Put $\phi = \sum_{j=1}^m \alpha_j h_j$ then ϕ belongs to $H^\infty(D^2)$ and $S_\phi = S_\psi$. □

Theorem 2.2. *Suppose $M = \{f \in H^2(D^2) : f = 0$ on $E\}$ for a subset E of D^2 and $\rho_2(a) = 0$ for any a in E. If ψ is a function in $H^2(D^2)$ such that S_ψ is of finite rank on N, then there exists a function ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi = 0$.*

Proof. If $S_\psi = 0$, put $\phi \equiv 0$ then $S_\psi = S_\phi$ and $S_\phi = 0$. Hence we assume $S_\psi \neq 0$. By the proof of Theorem 2.1, there exist minimal polynomials $p = p(z)$ and $q = q(w)$ which are one variable polynomials of degree $\ell \leq n$ such that $p\psi f \in M$ and $q\psi g \in M$ for any f and g in $N \cap H^\infty(D^2)$. Then the zeros of p (and q) are all simple and the degree of p (and q) is minimal, respectively. By the proof of Theorem 2.1, there exists $(a_1, b_1), \cdots, (a_m, b_m)$ in E such that $\psi = 0$ on $E \backslash \bigcup_{j=1}^{m}(a_j, b_j)$ and $p(a_j) = 0$ or $q(b_j) = 0$ $(1 \le j \le m)$. Let $p_t = p/(z - a_t)$ and $q_t = q/(z - b_t)$, then by the minimality of p and q there exist f_0 and g_0 in $N \cap H^\infty(D^2)$ such that $p_t\psi f_0 \notin M$ and $q_t\psi g_0 \notin M$. Hence $p_t\psi \notin M$ and $q_t\psi \notin M$ because $zM \subseteq M$ and $wM \subseteq M$. Since $\psi = 0$ on $E \backslash \bigcup_{j=1}^{m}(a_j, b_j)$ and $p_t(a_j) \ne 0$ or $q_t(b_j) \ne 0$ for $j \ne t$, $p_t\psi = q_t\psi = 0$ on $E \backslash (a_t, b_t)$. Hence $(p_t\psi)(a_t, b_t) \ne 0$ and $(q_t\psi)(a_t, b_t) \ne 0$ because $p_t\psi \notin M$ and $q_t\psi \notin M$. This contradicts $\rho_2(a) = 0$ for any a in E. □

When E does not have any isolated points, if ψ is a function in $H^2(D^2)$ such that S_ψ is of finite rank on N then there exists a function ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi = 0$.

Suppose $M = \{f \in H^2(D^2) : f = 0 \text{ on } E\}$. Then it is known [5, Theorem 4.1.1] that there exists an example $E \subset D$ such that $M \cap H^\infty(D^2) = \{0\}$. Then it is easy to see that there exists a in the example such that $\rho_2(a) > 0$ and $\rho_\infty(a) = 0$. When E is an interpolation sequence of $H^\infty(D^2)$, if $\rho_2(a) > 0$ then $\rho_\infty(a) > 0$ for any a in E.

It is easy to see that if there exists $a \in E$ such that $\rho_2(a) > 0$ then there exists ψ in $H^2(D^2)$ and $\dim S_\psi(N \cap H^\infty(D^2)) = 1$. Conversely if S_ψ is of rank one and ψ is in $H^2(D^2)$ then the proof of Theorem 2.1 shows there exists $a \in E$ such that $\rho_2(a) > 0$.

3. Problem 2

Let $\{a_j\}_{j=1}^n$ and $\{b_j\}_{j=1}^m$ be n and m distinct points in D, respectively. Put $\zeta_{ij} = (a_i, b_j)$ for $1 \le i \le n$ and $1 \le j \le m$. Then $\zeta_{ij} \ne \zeta_{\ell k}$ if $(i, j) \ne (\ell, k)$. Let $\{u_j\}_{j=1}^n$ and $\{v_j\}_{j=1}^m$ be in $\mathbb{C}$. Put $w_{ij} = u_i v_j$, $x = \max u_i$ and $y = \max v_j$ with $xy \le 1$. Under these notations, it is easy to see the following simple result by the original Pick's theorem for the disc.

Let Λ be a set $\{(i, j) : 1 \le i \le n, 1 \le j \le m\}$. There exist f in $H^\infty(D, z)$ with $\|f\|_\infty \le x$ and g in $H^\infty(D, w)$ with $\|g\|_\infty \le y$ such that $F(z, w) = f(z)g(w)$ and $F(\zeta_{ij}) = f(a_i)g(b_j) = u_i v_j = w_{ij}$ $((i, j) \in \Lambda)$ and $\|F\|_\infty \le 1$ if and only if

$$\left[\frac{x - u_s\bar{u}_t}{1 - \bar{a}_t a_s}\right]_{n\times m} \ge 0 \text{ and } \left[\frac{y - v_s\bar{v}_t}{1 - \bar{b}_t b_s}\right]_{m\times m} \ge 0.$$

4. Not necessary finite rank case

The condition in Theorem 4.1 satisfies one in Theorem 1. We use the generalized interpolation theorem of Sarason [6] for the proof. We consider a not necessary finite rank case. Moreover we consider 'Conjecture' in the previous paper [4]. Let $\mathcal{A}$ be the weak closed commutative Banach algebra generated by S_z, S_w and the identity operator and let $\mathcal{A}'$ denote the commutant of $\mathcal{A}$ in the Banach algebra of all bounded linear operators in N. 'Conjecture' is $\mathcal{A} = \mathcal{A}'$, that is, $\mathcal{A}' = \{S_\phi : \phi \in H^\infty(D^2)\}$.

Theorem 4.1. *Suppose $E = \bigcup_{j=1}^{\infty} E_{a_j\cdot}$ and $\{a_j\}$ is uniformly separated in D and $M = \{f \in H^2(D^2) : f = 0$ on $E\}$. If S_ψ is bounded and ψ is in $H^2(D^2)$ then for any finite p there exists a function ϕ in $H^p(D^2)$ such that $S_\psi = S_\phi$.*

Proof. By hypothesis, $E = \bigcup_{j=1}^{\infty} E_{a_j\cdot}$ and $\{a_j\}$ is uniformly separated in D. Since $\psi \in H^2(D^2)$, $\psi_j(w) = \psi(a_j, w) \in H^2(D, w)$. If $E_{a_j\cdot}$ does not satisfy a Blaschke condition then $E_{a_j\cdot} = \{a_j\} \times \bar{D}$ and so $M \subset (z - a_j)H^2(D^2)$. For if f is in M then $f(a_j, w) = 0$ on $E_{a_j\cdot}$ and so $f(a_j, w) \equiv 0$ on $\bar{D}$. Hence $N \supset (1 - \bar{a}_j z)^{-1} H^2(D, w)$ and so it is clear that $\psi_j(w) \in H^\infty(D, w)$ for $1 \le j \le m$. Then put $\phi_j = \psi_j$. If $E_{a_j\cdot}$ satisfies a Blaschke condition then $E_{a_j\cdot} = \{a_j\} \times \{b_{j1}, b_{j2}, \cdots\}$ and $\sum_{\ell=1}^{\infty}(1 - |b_{j\ell}|) < \infty$. Put $M_j = \{f \in H^2(D^2) : f(a_j, b_{j\ell}) = 0$ for $\ell = 1, 2, \cdots\}$ then $M_j \supset M$ and $N_j \subset N$ if $N_j = H^2(D^2) \ominus M_j$. Moreover N_j is the closed linear span of

$$\left\{ \frac{c_\ell}{(1 - \overline{a_j} z)(1 - \overline{b_{j\ell}} w)} : \ell = 1, 2, \cdots \right\}.$$

Hence $S_\psi^* \mid N_j = S_{\psi_j}^* \mid N_j$ and so there exists a $\phi_j \in H^\infty(D, w)$ such that $\psi_j - \phi_j \in M_j$ by the theorem of Sarason.

Considering $H^p(D^2) \supset H^\infty(D, H^p)$ for any $1 \le p < \infty$, the vector valued Hardy space on D, we can apply a theorem of Aron, Globevnik and Schottenloher [3] because $\{a_j\}$ is uniformly separated. There exists a function $\tilde{\phi} \in H^\infty(D, H^p)$ such that $\tilde{\phi}(a_j) = \phi_j(w)$ $(j = 1, 2, \cdots)$. Since we can write $\tilde{\phi}(z) = \phi(z, w) \in H^p(D^2)$, $\phi(a_j, w) = \phi_j(w)$ $(j = 1, 2, \cdots)$. Then $\psi - \phi$ belongs to M and so $S_\psi = S_\phi$. □

If $q_1 = q_1(z)$ and $q_2 = q_2(w)$ are one variable Blaschke products with the zero sets $\{a_j\}$ and $\{b_j\}$, respectively. Moreover if we assume that $a_i \ne a_j$ $(i \ne j)$ and $b_i \ne b_j$ $(i \ne j)$ then $M = q_1 H^2(D^2) + q_2 H^2(D^2) = \{f \in H^2(D^2) : f(a_j, b_j) = 0 \ 1 \le j < n\}$ where n may be ∞. Hence $N = (H^2(D, z) \ominus q_1 H^2(D, z)) \otimes (H^2(D, w) \ominus q_2 H^2(D, w))$. Now we can apply Theorem 4.1 for 'Conjecture' in [4].

5. Remark

In this section, we will discuss about the referee comments about Theorems 2.1 and 2.2. Let $E_2 = \{a \in E : \rho_2(a) > 0\}$ and $E_\infty = \{a \in E : \rho_\infty(a) > 0\}$, and $Z(\psi) = \{\lambda \in D^2 : \psi(\lambda) = 0\}$ for ψ in $H^2(D^2)$. He suggests the following (1)∼(5). (2) of the comment shows the converse of Theorem 2.1.

(1) There is ψ in $H^2(D^2)$ such that S_ψ is a nonzero operator of finite rank if and only if $E_2 \ne \emptyset$:

If there exists ψ in $H^2(D^2)$ which is of finite rank $n \ne 0$ then $E_2 \ne \emptyset$. This is just Theorem 2.2. However we could not show the converse is true or not. If $\rho_2(a) > 0$ for some $a \in E$, let ψ be a function in $H^2(D^2)$ such that $\psi(a) \ne 0$ and $\psi = 0$ on $E \backslash \{a\}$. Then there exists f in $N \cap H^\infty(D^2)$ such that $(\psi f)(a) \ne 0$ and $\psi f = 0$ on $E \backslash \{a\}$. Hence $\dim S_\psi(N \cap H^\infty(D^2)) = 1$. But we could not show S_ψ is bounded. Hence S_ψ may not be of finite rank.

(2) For every S_ψ of finite rank there exists a ϕ in $H^\infty(D^2)$ such that $S_\psi = S_\phi$ if and only if $E_2 = E_\infty$:

The ‘if’ part is clear by Theorem 2.1 and 2.2. We will show the ‘only if’ part. If S_ψ is of finite rank $n \neq 0$ by the proof of Theorem 2.1 $|\psi| > 0$ on $E\backslash Z(\psi) = \bigcup_{j=1}^m (a_j, b_j)$. Moreover for each $1 \leq j \leq m$, there exists a $f_j \in H^\infty(D^2)$ such that $f_j(a_j, b_j) = 1$ and $f_j(a_\ell, b_\ell) = 0$ for $j \neq \ell$. Put $\psi_j = f_j \psi$ then $\psi_j(a_j, b_j) = \psi(a_j, b_j) \neq 0$ and so $\rho_2((a_j, b_j)) > 0$. Since $S_{\psi_j} = S_{f_j} S_\psi$ because $S_z S_\psi = S_\psi S_z$ and $S_w S_\psi = S_\psi S_w$, S_{ψ_j} is bounded. By hypothesis, there exists ϕ_j in $H^\infty(D^2)$ such that $\psi_j - \phi_j \in M$ and so $\psi_j = \phi_j$ on E. Therefore $\phi_j(a_j, b_j) \neq 0$ and $\phi_j = 0$ on $E\backslash(a_j, b_j)$. Hence $\rho_\infty((a_j, b_j)) > 0$. Therefore $E_2 = E_\infty$.

(3) S_ψ is a nonzero operator of finite rank if and only if $E\backslash Z(\psi)$ is a finite set :

If S_ψ is of finite rank $n \neq 0$ then by the proof of Theorem 2.1 $\psi = 0$ on $E\backslash \bigcup_{j=1}^m (a_j, b_j)$ and so $E\backslash Z(\psi)$ is a finite set. For the converse, even if $E\backslash Z(\psi) = (a_1, b_1)$ by the same reason to (1) we could not show S_ψ is of finite rank.

(4) When S_ψ is bounded, the rank n of S_ψ equals to the number m of elements in $E\backslash Z(\psi)$:

This is clear for $n = 0$. When $n = 1$, the proof of Theorem 2.1 and the last three lines of the section 2 show (4) because S_ψ is bounded. In general, by the proof of Theorem 2.1, it is easy to see $n \leq m$. However it seems to be $m > n$.

(5) Let S_ψ be of finite rank. Then there is a $\phi \in H^\infty(D^2)$ such that $S_\psi = S_\phi$ if and only if $E_2\backslash Z(\psi) = E_\infty\backslash Z(\psi)$:

The ‘if’ part is clear by Theorem 2.1 and 2.2. The ‘only if’ part is not true. In fact, if ψ is nonzero constant α and $S_\psi \neq 0$ then the rank of S_ψ is one. On the other hand, $E(\psi) = \emptyset$. In this case, $\phi = \psi$. Of course, there exists E such that $E_2 \neq E_\infty$.

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