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Remarks on Riesz sets

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§ 1. Introduction.

Let $\hat{G}$ be the dual group of a LCA group G. $M(G)$ denotes the usual Banach algebra of all bounded regular Borel measures on G. $L^1(G)$ is the space of all integrable functions on G with respect to a Haal measure on G. For a $\mu\in M(G)$, its Fourier-Stieltjes transform $\hat{\mu}$ is defined as follows.

$$\hat{\mu}(\gamma)=\int_G(-x, \gamma)\, d\mu(x) \qquad \text{for } \gamma\in\hat{G}.$$

For a subset E of $\hat{G}$, $M_E(G)$ denotes the subspace of $M(G)$ consisting of measures whose Fourier-Stieltijes transforms vanish off E. Let $G=T^n$, and let P be a positive octant of $\hat{G}=Z^n$. Thar is $P=\{(m_1, \cdots, m_n)\in Z^n;\ m_i\geqq 0\ (i=1, \cdots, n)\}$. The following theorem (A) is called the Bochner's theorem.

(A) For every $\mu\in M_P(T^n)$, μ is absolutely continuous with respect to a Lebesque measure on T^n. That is, $M_P(T^n)\subset L^1(T^n)$.

If we exchange T^n by R^n, the same result is established.

The author proved in ([2]) the following theorem.

(B) Let G be a LCA group such that $\hat{G}$ is algebraically ordered. Let $M^a(G)$ denote the subspace of $M(G)$ consisting of measures of analytic type. Suppose $M^a(G)\neq\{0\}$. If $M^a(G)\subset L^1(G)$, then G admits one of the following structures.

(a) $G=R$, (b) $G=R\oplus D$,

(c) $G=T$, (d) $G=T\oplus D$

for some discrete abelian group D. Moreover, let G be one of the above groups. P is a subsemigroup of $\hat{G}$ such that (i) $P\cup(-P)=\hat{G}$ and $P\cap(-P)=\{0\}$. Set $M_P^a(G)=M^a(G)$. Then, $M_P^a(G)\subset L^1(G)$.

We start to consider whether an analogy of the Bochner's theorem is established if we exchange T by $T\oplus D$.

PROPOSITION 1. *Let $H=T\oplus D$, where D is a discrete abelian group such that $\hat{D}$ is torsion-free. Let P_H be a subsemigroup of $\hat{H}=Z\oplus\hat{G}$ such that*

(i) $P_H \cup (-P_H) = \hat{H}$ *and* (ii) $P_H \cap (-P_H) = \{0\}$. *Let* $G = H^n (= \overbrace{H \oplus \cdots \oplus H}^{n})$, then $M_{P_H^n}(G)$ is included in $L^1(G)$, where $P_H^n = \overbrace{P_H \times \cdots \times P_H}^{n} = \{(\gamma_1, \cdots, \gamma^n) \in \hat{G}\,;\ \gamma_i \in P_H,\ i = 1, 2, \cdots, n\}$.

In order to prove this proposition, we need the following two lemmas.

LEMMA 1. *Let F be a compact torsion-free abelian group.*

(1) *Let P be a subsemigroup of $Z \oplus F$ such that* (i) $P \cup (-P) = Z \oplus F$ *and* (ii) $P \cap (-P) = \{0\}$. *If P is not dense in $Z \oplus F$, then*

$$P = \Big\{(n, f) \in Z \oplus F\,;\ n > 0,\ \text{or}\ n = 0\ \text{and}\ f \geqq_P 0\Big\}$$

or

$$= \Big\{(n, f) \in Z \oplus F\,;\ n < 0,\ \text{or}\ n = 0\ \text{and}\ f \geqq_P 0\Big\}$$

(2) *Let P be a subsemigroup of $R \oplus F$ such that* (i) $P \cup (-P) = R \oplus F$ *and* (ii) $P \cap (-P) = \{0\}$. *If P is not dense in $R \oplus F$, then*

$$P = \Big\{(x, f) \in R \oplus F\,;\ x > 0,\ \text{or}\ x = 0\ \text{and}\ f \geqq_P 0\Big\}$$

or

$$= \Big\{(x, f) \in R \oplus F\,;\ x < 0,\ \text{or}\ x = 0\ \text{and}\ f \geqq_P 0\Big\}$$

Where '$<$' denotes the usual order on Z and R, and '$\geqq_P$' denotes the order on F induced by P.

PROOF. We prove only (2). Suppose P is not dense in $R \oplus F$. Since P is dense in F, $P \cap R$ is not dense in R. Hence, by proposition 2 of [3], $P \cap R = [0, \infty)$ or $(-\infty, 0]$. We consider only the case $P \cap R = [0, \infty)$. Suppose there exists an $x_0 > 0\,(x_0 \in R)$ such that $(x_0, f_0) \in (\overline{-P})$ for some $f_0 \in F$. Then $\{(x, f) \in R \oplus F\,;\ x \leqq x_0,\ f \in F\}$ is included in $(\overline{-P})$. Since $(\overline{-P})$ is a semigroup, $(\overline{-P}) = R \oplus F$. That is, P is dense in $R \oplus F$. This is a contradiction. Q. E. D.

DEFINITION 1. *G is a LCA group. A subset E of $\hat{G}$ is called the Riesz set if $M_E(G) \subset L^1(G)$.*

LEMMA 2. *Let G_1 be a LCA group, and let G_2 be a discrete abelian group. If $E (\subset \hat{G}_1)$ is a Riesz set of $\hat{G}_1$, then $E \times \hat{G}_2$ is a Riesz set of $\hat{G}_1 \oplus \hat{G}_2$.*

PROOF. For $\mu \in M_{E \times \hat{G}_2}(G_1 \oplus G_2)$, μ is represented as follows.

$$d\mu(x, y) = d\mu_{1,n}(x) \times d\delta_{y_n}(y)\ \Big((x, y) \in G_1 \oplus G_2\Big)$$

, where $\mu_{1,n}$ belongs to $M(G_1)$ and δ_{y_n} is the Dirac measure at $y_n \in G_2$ ($n=1, 2, \cdots$), and moreover $\|\mu\| = \sum_{n=1}^{\infty} \|\mu_{1,n}\|$ For $\gamma_1 \notin E$, $(\gamma_1, \gamma_2) \notin E \times \hat{G}_2$ for every $\gamma_2 \in \hat{G}_2$.

Hence,

$$\sum_{n=1}^{\infty} \hat{\mu}_{1,n}(\gamma_1)\,(-y_n, \gamma_2)$$
$$= \hat{\mu}(\gamma 1, \gamma_2) = 0 \qquad \text{for every } \gamma_2 \in \hat{G}_2.$$

Since $\sum_{n=1}^{\infty} |\hat{\mu}_{1,n}(\gamma_1)| < \infty$, we can derive that $\hat{\mu}_{1,n}(\gamma_1)=0$ ($n=1, 2, \cdots$). That is, $\mu_{1,n} \in M_E(G_1)$ ($n=1, 2, \cdots$). By the hypothesis, $\mu_{1,n}$ belongs to $L^1(G_1)$, and so μ is contained in $L^1(G_1 \oplus G_2)$. Q. E. D.

Proof Of Proposition 1. If P_H is dense in $\hat{H}$, then P_H^n is also dense in $\hat{G}$. Hence, $M_{P_H^n}(G) = \{0\} \subset L^1(G)$. If P_H is not dense in $\hat{H}$, then by lemma 1, $P_H \subset Z^+ \times \hat{D}$, where Z^+ is a subset of Z consisting of nonnegative integers.

Hence, $P_H^n \subset \overbrace{(Z^+ \times \hat{D}) \times \cdots \times (Z^+ + \hat{D})}^{n} \cong (Z^+)^n \times \hat{D}_n$.

By the Bochner's theorem, $(Z^+)^n$ is a Riesz set in Z^n, and $\hat{D}^n$ is a compact abelian group. Hence, by lemma 1, we obtain that $M_{P_H^n}(G) \subset L^1(G)$.

Q. E. D.

Remark 1. *The same result is established in proposition* 1 *even if we exchange $T \oplus D$ by $R \oplus D$.*

Combing with theorem 1 and theoren 2 of [2], the following corollary is obtained.

Corollary 1. *Let G be a LCA group. Let P be a subsemigroup of $\hat{G}$ such that* (i) $P \cup (-P) = \hat{G}$ *and* (ii) $P \cap (-P) = \{0\}$. *Suppose P is not dense in $\hat{G}$. Then, the following are equivalent.*

(1) $M_{P^n}(G^n) \subset L^1(G^n)$.

(2) *G admits one of the following structures.*

(a) $G = R$, (b) $G = \mathrm{R} \oplus \mathrm{D}$, (c) $G = T$, (d) $G = T \oplus D$ *for some discrete abelian group D.*

§ 2. Small p sets

In this section, we shall prove that the direct product of a small p set and a compact set is a small p set.

Definition 2. *G is a LCA group. Let p be a positive integer. A subset E of $\hat{G}$ is said to be a small p set if the following property is satisfied.* (*See* [5]).

(*) *For* $\mu \in M_E(G)$, $\mu^p = \overbrace{\mu * \cdots * \mu}^{p}$ *belongs to* $L^1(G)$.

LEMMA 3. *Let p be a positive integer. Then,*

$$x_1 x_2 \cdots x_p = \sum_{j=1}^{C(p)} \alpha_j \left\{\beta_j(x_1, x_2, \cdots, x_p)\right\}^p$$

for every complex numbers x_i ($i=1, 2, \cdots, p$), where β_j are linear forms of $x_1, x_2, \cdots$, and x_p, α_j are real numbers ($j=1, 2, \cdots, p$) and $C(p)$ is a positive integer.

PROOF.

$$4x_1 x_2 = (x_1+x_2)^2 - (x_1-x_2) \tag{1}$$

We integrate two side of (1) with respect to x_1 from x_1 to $2x_1$, then

$$x_1^2 x_2 = \frac{1}{9}\left\{(2x_1+x_2)^3 - (x_1+x_2)^3 - (2x_1-x_2)^3 + (x_1-x_2)^3\right\} \tag{2}$$

We integrate two sides of (2) with respect to x_1 from x_1 to $2x_1$ again, then

$$\begin{aligned} x_1^3 x_2 = \frac{1}{168}\Big\{&(4x_1+x_2)^4 - (2x_1+x_2)^4 - 2(2x_1+x_2)^4 \\ &+ 2(x_1+x_2)^4 - (4x_1-x_2)^4 + (2x_1-x_2)^4 \\ &+ 2(2x_1-x_2)^4 - 2(x_1-x_2)^4\Big\}. \end{aligned}$$

We continue this argument. Then, for each positive integer n, there exist linear forms $A_{n,i}(x_1, x_2)$ and real numbers $a_{n,i}$ ($i=1, 2, \cdots, 2^n$) such that

$$x_1^n x_2 = \sum_{i=1}^{2^n} a_{n,i}\left\{A_{n,i}(x_1, x_2)\right\}^{n+1}.$$

We define linear forms $B_{i_1,\cdots,i_k}(x_1, \cdots, x_k, x_{k+1})$ ($k=1, 2, \cdots$; $1 \leqq i_j \leqq 2^j$, $j= 1, 2, \cdots, k$.) as follows.

$$B_{i_1}(x_1, x_2) = A_{1,i_1}(x_1, x_2),$$

$$B_{i_1,i_2}(x_1, x_2, x_3) = A_{2,i_2}\left(B_{i_1}(x_1, x_2), x_3\right),$$

$$B_{i_1,i_2,i_3}(x_1, x_2, x_3, x_4) = A_{3,i_3}\left(B_{i_1,i_2}(x_1, x_2, x_3), x_4\right),$$

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$$B_{i_1,\cdots,i_k}(x_1, \cdots, x_k, x_{k+1}) = A_{k+1,i_{k+1}}\left(B_{i_1,\cdots,i_{k-1}}(x_1, \cdots, x_k), x_{k+1}\right)$$

Then, we obtain the following equality.

$$x_1 x_2 \cdots x_p = \sum_{i_1=1}^{2} \cdots \sum_{i_{p-1}=1}^{2^{p-1}} a_{1,i_1} \cdots a_{p-1,i_{p-1}} \left\{ B_{i_1,\cdots,i_{p-1}}(x_1, \cdots, x_p) \right\}^p .$$

Q. E. D.

LEMMA 4. *Let G be a LCA group. Let E be a subset of $\hat{G}$. Then, the following are equivalent.*

(1) *E is a small set,*

(2) *$\mu_1 * \cdots * \mu_p$ belong to $L^1(G)$ for every $\mu_1, \cdots, \mu_0 \in M_E(G)$.*

PROOF. (2)$\Rightarrow$(1). trivial.

(1)$\Rightarrow$(2). For $\mu_1, \cdots, \mu_p \in M_E(G)$, by lemma 3,

$$\mu_1 * \cdots * \mu_p = \sum_{j=1}^{C(P)} \alpha_j \overbrace{\beta_j(\mu_1, \cdots, \mu_p) * \cdots * \beta_j(\mu_1, \cdots, \mu_p)}^{P} .$$

Since $\beta_j(\mu_1, \cdots, \mu_p) \in M_E(G)$, $\mu_1 * \cdots * \mu_p$ belongs to $L^1(G)$. Q. E. D.

Lemma 2 of §1 can be generalized as follows.

PROPOSITION 5. *Let G be a LCA group and F a compact subgroup of $\hat{G}$. Let π_F be a natural homomorphism from $\hat{G}$ onto $\hat{G}/F$. If $\check{E}$ is a small p subset of $\hat{G}/F$, then $\pi_F^{-1}(\check{E})$ is also a small p set in $\hat{G}$.*

PROOF. Let H be an annihilator of F. Since F is compact, H is an open subgroup of G. Let μ_{H_α} be a restriction of μ to each cosets H_α of H. Then, μ can be represented as follows.

$$\mu = \sum_{n=1}^{\infty} \mu_{H+x_n}$$

, where $H + x_n \neq H + x_m$ if $n \neq m$, and $\|\mu\| = \sum_{n=1}^{\infty} \|\mu_{H+x_n}\|$.

Set $\lambda_n = \mu_{H+x_n} * \delta_{-x_n}$ $(n=1, 2, \cdots)$, where δ_{-x_n} is the Dirac measure at $-x_n$. Then, $\lambda_n \in M(H)$, and $\mu = \sum_{n=1}^{\infty} \delta_{x_n} * \lambda_n$.

For $\gamma \notin \pi_F^{-1}(\check{E})$, $\gamma + s \notin \pi_F^{-1}(\check{E})$ for every $s \in F$.

Hence,

$$\begin{aligned} &\sum_{n=1}^{\infty} \hat{\lambda}_n\left(\pi_F(\gamma)\right)(-x_n, s) \\ &= \sum_{n=1}^{\infty} \hat{\lambda}_n(\gamma + s)(-x_n, \gamma + s) \\ &= \hat{\mu}(\gamma + s) \\ &= 0 \qquad \text{for every } s \in F . \end{aligned}$$

Since $\sum_{n=1}^{\infty} |\hat{\lambda}_n(\pi_F(\gamma))| < \infty$ and F is dense in its Bohr compacti fication $\bar{F}^B$, $\hat{\lambda}_n(\pi_f(\gamma))(-x_n, \gamma) = 0$ $(n=1, 2, \cdots)$.

That is, $\lambda_n \in M^{\check{F}}(H)$ $(n=1, 2, \cdots)$. On the other hand,

$$\mu^p = \sum_{i_1=1}^{\infty} \cdots \sum_{i_p=1}^{\infty} (\lambda_{i_1} * \cdots * \lambda_{i_p}) * \delta_{x_{i_1} + \cdots + x_{i_p}} .$$

Hence, by lemma 4, μ^p belongs to $L^1(G)$. Q. E. D.

LEMMA 6. *Let G be a LCA group, and let Λ be an open subgroup of $\hat{G}$. E is a subset of Λ. Then, E is a small p set in $\hat{G}$ if and only if E is a small p set in Λ.*

PROOF. Suppose E is a small p set in Λ. For a $\mu \in M_E(G)$, there exists a measure $\lambda \in M_E(G/\Lambda^\perp)$ such that $\hat{\mu}|_\Lambda = \hat{\lambda}$, where $\Lambda^\perp$ is an annihilator of Λ.

Since $\widehat{L^1(G)}|_\Lambda = \widehat{L^1(G/\Lambda^\perp)}$, there exists a function $g \in L^1(G)$ such that $\hat{g}|_\Lambda = \hat{\lambda}^n$. Let $m_{\Lambda^\perp}$ be a normalized Haar measure on $\Lambda^\perp$. Then, $\mu^p = m_{\Lambda^\perp} * g \in L^1(G)$.

Conversely, if E is a small p set in $\hat{G}$, for a $\lambda \in M_E(G/\Lambda^\perp)$, there exists a measure $\mu' \in M(G)$ such that $\hat{\mu}'|_\Lambda = \hat{\lambda}$.

Let $\mu = \mu' * m_{\Lambda^\perp}$. Then, μ belongs to $M_E(G)$, and so, by the hypothesis, μ^p is absolutely continuous with respect to a Haar measure on G. Hence, by Theorem 2. 7. 4 of [6], λ^p belongs to $L^1(G/\Lambda^\perp)$. Q. E. D.

LEMMA 7. *Let G_1 be a LCA group, and let G_2 be a compact abelian group. Let p be a positive integer. A subset E_1 of $\hat{G}_1$ is a small p set in $\hat{G}_1$. E_2 is a compact subset. Then, $E_1 \times E_2$ is a small p set in $\widehat{G_1 \oplus G_2}$.*

PROOF. Set $E_2 = \{\gamma_1', \gamma_2', \cdots, \gamma_n'\}$. Let μ be a measure belonging to $M_{E_1 \times E_2}(G_1 \oplus G_2)$. For each k $(k=1, 2, \cdots, n)$, define a continuous function ψ_k on $\hat{G}_1$ as follows.

$$\psi_k(\gamma) = \hat{\mu}(\gamma, \gamma_k') .$$

Then, by Theorem 1. 9. 1 of [6], there exists a measure $\mu_k \in M_{E_1}(G_1)$ such that $\hat{\mu}_k(\gamma) = \psi_k(\gamma)$. Hence, $\mu = \sum_{k=1}^{n} \mu_k \times (y, \gamma_k')\, m_{G_2}$, where m_{G_2} is a normalized Haar measure on G_2. Since E_1 is a small p set,

$$\overbrace{\mu * \cdots * \mu}^{p} = \sum_{k_1, \cdots, k_p = 1}^{n} \mu_{k_1} * \cdots * \mu_{k_p} \times (\gamma_{k_1}' m_{G_2}) * \cdots * (\gamma_{k_p}' m_{G_2})$$
$$= \sum_{k=1}^{n} \mu_k^p \times (\gamma_k' m_{G_2}) \in L^1(G_1 \oplus G_2) . \qquad \text{Q. E. D.}$$

LEMMA 8. *Let G be a LCA group and p be a positive intiger.*

Let E be a small p subset of $\hat{G}$, and let F be a compact subset of R^n. Then, $E \times F$ is a small p set in $\widehat{G \oplus R^n}$.

PROOF. For $\mu\in M_{E\times F}(G\oplus R^n)$, let $(\mu^p)_s$ be a singular part of $\mu^p=\overbrace{\mu*\cdots*\mu}^{p}$ with respect toa Haar measure on $\hat{G}$. Suppose $(\mu^p)_s\neq 0$. For a positive number c, we define a homeomorphism I_C on $G\oplus R^n$ as follows.

$$I_C(x, y)=(x, cy) \qquad \text{for } (x, y)\in G\oplus R^n$$

, where $y=(y_1, y_2, \cdots, y_n)\in R^n$ and $cy=(cy_1, cy_2, \cdots, cy_n)$.

For a measure $\lambda\in M(G\oplus R^n)$, $I_C\circ\lambda$ denotes the continuous image of λ under I_C. Since $I_C\circ\mu^p=(I_C\circ\mu)^p$ and μ is regular, we may hypothesize that

$$\left|(\mu^p)_s\right|\left(G\times(-\pi, \pi]^n\right)>\frac{1}{2}\left\|(\mu^p)_s\right\| .$$

Let ϕ be a natural homomorphism from $G\oplus R^n$ onto $G\oplus T^n$.

That is, $\phi(x, y)=(x, e^{iy})$ for $(x, y)\in G\oplus R^n$. Then, $\phi\circ(\mu^p)_s$ is also a singular measure on $G\oplus T^n$. And, since $|(\mu^p)_s|(G\times(-\pi, \pi]^n)>\frac{1}{2}\|(\mu^p)_s\|$, $\phi\circ(\mu^p)_s\neq 0$. Hence, $\phi\circ(\mu^p)_s$ (image of $(\mu^p)_s$ under ϕ) is a nonzero singular part of $\phi\circ(\mu^p)$ with respect to a Haar measure on $G\oplus T^n$.

On the other hand, since F is a compact subset of R^n, there exists a positive integer m_0 such that $F\subset C_{m_0}$,

$$C_{m_0}=\left\{x=(x_1, x_2, \cdots, x_n)\in R^n\,;\ |x_i|\leqq m_0\,,\quad i=1, 2, \cdots, n\right\}.$$

For $s\in\hat{G}$ and $k=(k_1, \cdots, k_n)\in Z^n$, $\widehat{\phi\circ(\mu^p)}(s, k)=\hat{\mu}^p(s, k)$.

That is, $\operatorname{supp}(\widehat{\phi\circ(\mu^p)})$ is included in $E\times(Z^n\cap C_{m_0})$. Hence, by lemma 7, $\phi\circ(\mu^p)$ belongs to $L^1(G\oplus T^n)$. This is a contradiction. Q. E. D.

THEOREM 9. *Let G_1 and G_2 be LCA groups. Let p be a positive integer. If E is a small p subset of $\hat{G}_1$ and F is a compact subset of $\hat{G}_2$, then $E\times F$ is a small p set in $\widehat{G_1\oplus G_2}$.*

PROOF. Let F_1 be a compact symmetric neighbourhood of 0 including F. Let F_0 be an open subgroup of $\hat{G}_2$ generated by F_1. That is, $F_0=\bigcup_{n=1}^{\infty}(\overbrace{F_1+\cdots+F_1}^{n})$. Then, by Theorem 9.8 of [1], $F_0\cong R^m\oplus Z^n\oplus F^*$, where m and n are nonnegative integers, and F^* is a compact abelian group. By lemma 6, we may show that $E\times F$ is a small p set in $\hat{G}_1\oplus F_0=\hat{G}_1\oplus R^m\oplus Z^n\oplus F^*$.

Let K_1 be a projection of F to R_m, and let K_2 be a projection of F to Z^n. Then, K_1 and K_2 are compact subsets. Hence, by lemma 8, $E\times K_1$ is a small p set in $\hat{G}_1\oplus R^m$, and so, by lemma 7, $E\times K_1\times K_2$ is a small p set in $\hat{G}_1\oplus R^m\oplus Z^n$.

Therefore, by proposition 5, $E\times K_1\times K_2\times F^*$ is a small p set in $\hat{G}_1\oplus F_0$. Since $E\times F$ is included in $E\times K_1\times K_2\times F^*$, $E\times F$ is a small p set in $\hat{G}_1\oplus F_0$.

Q. E. D.

DEFINITION 3. *Let G be a LCA group, and let E be a subset of $\hat{G}$. E is called a strong Riesz set if its closure with respect to the relative topology of Bohr compactification of $\hat{G}$.* (*See* [3]).

COROLLARY 2. *Let G_i be LCA groups* (i=1, 2). *If E_1 is a strong Riesz set of $\hat{G}_1$ and E_2 is a compact subset of $\hat{G}_2$. Then, $E_1\times E_2$ is a strong Riesz in $\widehat{G_1\oplus G_2}$.*

PROOF. Since E_2 is compact, $\overline{E_1\times E_2}^{\mathscr{F}}$ is included in $\bar{E}_1^{\mathscr{F}}\times E_2$, where '$-\mathscr{F}$' denotes the closure with respect to the relative topology induced by the topology of the Bohr compactification.

Hence, by theorem 9, the conclusion is obtained. Q. E. D.

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