ON SOME PROPERTIES OF CERTAIN HYPERSURFACES IN A K-SPACE

By

Hidemaro KÔJYÔ

Introduction. Recently Y. Tashiro [5][1)] proved that an orientable hypersurface in an almost complex space has an almost contact structure and showed that the induced almost contact structure of the hypersurface in a Kählerian space is normal contact if and only if the second fundamental tensor of the hypersurface has the form $H_{\alpha\beta}=g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta$ [5].

The purpose of the present paper is to investigate some properties of a hypersurface with analogous conditions in a K-space. § 1 devoted to give the fundamental concepts of an almost Hermitian space, and we show some formulas concerning hypersurfaces in a K-space for the later use.

It is well-known that if the second fundamental tensor of a hypersurface in Euclidean space has the form $H_{\alpha\beta}=g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta$, then $\mu=0$, that is, the hypersurface is totally umbilical [6], [7]. In § 2, we shall obtain the similar properties of such a hypersurface in the special K-space. In the last section we consider of the case that a hypersurface in a K-space admits the second fundamental tensor of more general form $H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta$, and we shall give properties of such a hypersurface in the special K-space.

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§ 1. Preliminaries. Let us consider a real $(2n+2)$-dimensional almost Hermitian manifold M^{2n+2} with local coordinate system $\{x^i\}$ and let $(F^i{}_j,\ g_{ij})$ be the almost Hermitian structure, that is, $F^i{}_j$ be the almost complex structure defined on M^{2n+2} and g_{ij} be the Riemannian metric tensor satisfying $g_{hk}=g_{ij}F^i{}_hF^j{}_k$. Then it follows that

$$(1.1)\qquad F_{ij}=-F_{ji}\,. \qquad (F_{ij}=g_{ih}F^h{}_j)$$

A differentiable hypersurface M^{2n+1} of M^{2n+2} may be represented parametrically by the equation $X^i=X^i(u^\alpha)$[2)]. If we put

1) Numbers is brackets refer to the references at the end of the paper.

2) Throughout the present paper the Latin indices are supposed to run over the range $1, 2, \cdots, 2n+2$, and the Greek indices take the values $1, 2, \cdots, 2n+1$.

$$X_\alpha^i = \frac{\partial x^i}{\partial u^\alpha},$$

X_α^i span a tangent plane of M^{2n+1} at each point, and the induced Riemannian metric tensor $g_{\alpha\beta}$ in M^{2n+1} is given by

$$g_{\alpha\beta} = g_{ij} X_\alpha^i X_\beta^j .$$

Assuming that our hypersurface is orientable, we choose the unit normal vector X^i to the hypersurface and put

$$(1.2) \qquad \varphi^\alpha{}_\beta = F^i{}_j X_i^\alpha X_\beta^j , \quad \xi^\alpha = -F^i{}_j X_i^\alpha X^j , \quad \eta_\alpha = F^i{}_j X_i X_\alpha^j ,$$

where we put $X_i^\alpha = g^{\alpha\beta} g_{ij} X_\beta^j$ and $X_i = g_{ij} X^j$.

Then it is known that the quantities $\varphi^\alpha{}_\beta$, ξ^α, η_α and $g_{\alpha\beta}$ satisfy the following conditions [3]:

$$(1.3) \qquad \begin{cases} \xi^\alpha \eta_\alpha = 1 , & \operatorname{rank}(\varphi^\alpha{}_\beta) = 2n , \\ \varphi^\alpha{}_\beta \xi^\beta = 0 , & \varphi^\alpha{}_\beta \eta_\alpha = 0 , \\ \varphi^\alpha{}_\beta \varphi^\beta{}_\gamma = -\delta^\alpha{}_\gamma + \xi^\alpha \eta_\gamma , & \end{cases}$$

and

$$(1.4) \qquad g_{\alpha\beta} \xi^\alpha = \eta_\beta , \quad g_{\alpha\beta} \varphi^\alpha{}_\gamma \varphi^\beta{}_\delta = g_{\gamma\delta} - \eta_\gamma \eta_\delta .$$

Therefore we may consider the quantities $\varphi^\alpha{}_\beta$, ξ^α, η_α and $g_{\alpha\beta}$ define an almost contact metric structure in M^{2n+1}. From (1.3) and (1.4) it follows that

$$(1.5) \qquad \varphi_{\alpha\beta} = -\varphi_{\beta\alpha} . \qquad (\varphi_{\alpha\beta} = g_{\alpha\gamma} \varphi^\gamma{}_\beta)$$

On making use of the Gauss equations we have from (1.2)

$$(1.6) \qquad \begin{cases} \varphi^\alpha{}_{\beta;\gamma} = F^i{}_{j,k} X_i^\alpha X_\beta^j X_\gamma^k + H^\alpha{}_\gamma \eta_\beta - H_{\beta\gamma} \eta^\alpha , \text{[3]} \\ \eta^\alpha{}_{;\beta} = -F^i{}_{j,k} X_i^\alpha X^j X_\beta^k + H^\gamma{}_\beta \varphi^\alpha{}_\gamma , \end{cases}$$

where $H^\alpha{}_\beta = g^{\alpha\gamma} H_{\beta\gamma}$ and $H_{\beta\gamma}$ denotes the covariant component of the second fundamental tensor of M^{2n+1}.

Let M^{2n+2} be a K-space, then we have the following condition [1]:

$$(1.7) \qquad F_{ij,k} + F_{ik,j} = 0 .$$

If we put $F_{ij,k} X_\alpha^i X_\beta^j X_\gamma^k = A_{\alpha\beta\gamma}$, then by virtue of (1.1) and (1.7) we can find that the covariant tensor $A_{\alpha\beta\gamma}$ is skew-symmetric with respect to all indices, and it follows that if $A_{\alpha\beta\gamma} = 0$ holds good, then a K-space is necessarily a Kählerian space [10]. Moreover, after some calculations (1.6) become

3) In the present paper comma and semi-colon denotes covariant differentiation with respect to the Riemannian connection defined by g_{ij} and its induced connection respectively.

$$(1.8) \qquad \varphi_{\alpha\beta;\gamma} = A_{\alpha\beta\gamma} + H_{\alpha\gamma}\eta_\beta - H_{\beta\gamma}\eta_\alpha ,$$

$$(1.9) \qquad \eta^\alpha{}_{;\beta} = H^\gamma{}_\beta \varphi^\alpha{}_\gamma + A^\varepsilon{}_{\delta\beta}\eta^\delta \varphi^\alpha{}_\varepsilon \ [10].$$

Then for the Nijenhuis tensors of a hypersurface in a K-space we obtain the following expressions:

$$(1.10) \qquad N_\alpha = H_{\beta\gamma}\varphi^\beta{}_\alpha \eta^\gamma ,$$

$$(1.11) \qquad N_{\beta\gamma} = 2A_{\alpha\beta\gamma}\eta^\alpha - 2A_{\alpha\delta\varepsilon}\eta^\alpha \varphi^\delta{}_\beta \varphi^\varepsilon{}_\gamma + H_{\alpha\gamma}\eta^\alpha \eta_\beta - H_{\alpha\beta}\eta^\alpha \eta_\gamma ,$$

$$(1.12) \qquad N^\alpha{}_\beta = 2A^\alpha{}_{\beta\gamma}\eta^\gamma + H^\alpha{}_\gamma \eta^\gamma \eta_\beta - H^\alpha{}_\beta - \varphi^\gamma{}_\beta \eta^\alpha{}_{;\gamma} ,$$

$$(1.13) \qquad N^\alpha{}_{\beta\gamma} = 2A^\alpha{}_{\beta\delta}\varphi^\delta{}_\gamma - 2A^\alpha{}_{\gamma\delta}\varphi^\delta{}_\beta + H^\alpha{}_\delta \eta_\beta \varphi^\delta{}_\gamma - H^\alpha{}_\delta \eta_\gamma \varphi^\delta{}_\beta + \eta^\alpha{}_{;\beta}\eta_\gamma - \eta^\alpha{}_{;\gamma}\eta_\beta .$$

§ 2. The hypersurface admitting the second fundamental tensor of the form $H_{\alpha\beta} = g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta$. In a K-space M^{2n+2}, the induced structure $\varphi_{\alpha\beta}$ in a hypersurface M^{2n+1} satisfies the relation

$$(2.1) \qquad \varphi_{\alpha\beta} = \eta_{\alpha;\beta} + A_{\gamma\delta\beta}\eta^\delta \varphi^\gamma{}_\alpha ,$$

if and only if the second fundamental tensor $H_{\alpha\beta}$ of M^{2n+1} has the form

$$(2.2) \qquad H_{\alpha\beta} = g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta ,$$

where μ is a scalar field in M^{2n+1} [10].

Now, if we assume that the manifold M^{2n+2} be a K-space with constant curvature, then the hypersurface satisfies the following Codazzi equation [2]:

$$(2.3) \qquad H_{\alpha\beta;\gamma} - H_{\gamma\beta;\alpha} = 0 .$$

Substituting (2.2) into (2.3), we get

$$(2.4) \qquad (\mu_{;\gamma})\eta_\alpha\eta_\beta - (\mu_{;\alpha})\eta_\beta\eta_\gamma + \mu(\eta_{\alpha;\gamma}\eta_\beta - \eta_{\gamma;\alpha}\eta_\beta + \eta_{\beta;\gamma}\eta_\alpha - \eta_{\beta,\alpha}\eta_\gamma) = 0 .$$

Multiplying (2.4) by $\varphi^{\alpha\gamma}\eta^\beta$ and summing for all indices, by virtue of (1.3) it follows that

$$(2.5) \qquad \mu\cdot\varphi^{\alpha\gamma}(\eta_{\alpha;\gamma} - \eta_{\gamma;\alpha}) = 0 .$$

By means of (2.1) and the skew-symmetric property of the tensor $A_{\alpha\beta\gamma}$, we obtain $\mu = 0$. Thus we have the following theorem:

Theorem 2.1. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in a K-space with constant curvature has the form*

$$H_{\alpha\beta} = g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta ,$$

then $\mu = 0$ and the hypersurface is umbilical.

When a K-space is an Einstein space, we obtain the following theorem:

Theorem 2.2. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface* *in an Einstein K-space is of the form*

$$H_{\alpha\beta}=g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta\,,$$

then $\mu=const.$ *and the mean curvature* H *of a hypersurface is constant.*

Proof. Substituting (2.1) and (2.2) into the Codazzi equation [2]:

$$H_{\alpha\beta;\gamma}-H_{\gamma\beta;\alpha}=K_{ijkl}X^i_\gamma X^j_\alpha X^k_\beta X^l\,, \tag{2.6}$$

we obtain

$$\begin{aligned}&(\mu_{;\gamma})\eta_\alpha\eta_\beta-(\mu_{;\alpha})\eta_\beta\eta_\gamma-\mu(2\eta_\beta\varphi_{\alpha\gamma}+\eta_\alpha\varphi_{\beta\gamma}-\eta_\gamma\varphi_{\beta\alpha})\\ &\qquad+\mu(A_{\varepsilon\delta\gamma}\eta^\varepsilon\eta_\beta\varphi^\delta{}_\alpha+A_{\varepsilon\delta\gamma}\eta^\varepsilon\eta_\alpha\varphi^\delta{}_\beta-A_{\varepsilon\delta\alpha}\eta^\varepsilon\eta_\beta\varphi^\delta{}_\gamma-A_{\varepsilon\delta\alpha}\eta^\varepsilon\eta_\gamma\varphi^\delta{}_\beta)\\ &=K_{ijkl}X^i_\gamma X^j_\alpha X^k_\beta X^l\,.\end{aligned} \tag{2.7}$$

Since M^{2n+2} is an Einstein K-space, multiplying (2.7) by $g^{\alpha\beta}$ and summing for α and β, we get

$$(\mu_{;\gamma})-(\mu_{;\alpha})\eta^\alpha\eta_\gamma-\mu A_{\beta\delta\alpha}\eta^\beta\eta_\gamma\varphi^{\delta\alpha}=0\,, \tag{2.8}$$

from which by multiplying (2.8) by $\varphi^{\gamma\varepsilon}\varphi_{\varepsilon\lambda}$ and summing for γ, we obtain

$$(\mu_{;\gamma})\eta^\gamma\eta_\lambda=\mu_{;\lambda}\,. \tag{2.9}$$

Moreover, differentiating (2.9) covariantly with respect to u^α and multiplying by $\varphi^{\alpha\lambda}$ and summing for λ, by virtue of (2.1) we have the following relation:

$$(\mu_{;\gamma})\eta^\gamma\eta_\lambda=0\,. \tag{2.10}$$

Hence μ is constant in M^{2n+1} from (2.9) and (2.10).

§3. The hypersurface admitting the second fundamental tensor of the form $H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta$. In this section, we consider that a hypersurface of a K-space admits the second fundamental tensor $H_{\alpha\beta}$ of the form

$$H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_\alpha\eta_\beta\,. \tag{3.1}$$

where λ and μ are scalar functions.

At the first, let us consider a relation of a hypersurface, which is equivalent to (3.1).

Substituting from (3.1) in the right hand member of (1.9), we have

$$\eta_{\alpha;\beta}=\lambda\varphi_{\alpha\beta}+A_{\varepsilon\delta\beta}\eta^\varepsilon\varphi^\delta{}_\alpha\,. \tag{3.2}$$

Next, we shall show that (3.2) is the sufficient condition that $H_{\alpha\beta}$ be of the form (3.1). Multiplying (3.2) by η^β and summing for β, we get $\eta_{\alpha;\beta}\eta^\beta=0$. From the last relation it follows that $N_\alpha=0$, then from (1.10) we obtain the relation

$$H_{\alpha\beta}\eta^{\beta}=\rho\eta_{\alpha}\,, \tag{3.3}$$

for a suitable function ρ.

Moreover, by using (1.8) we have

$$\varphi_{\alpha\gamma;\beta}+\varphi_{\beta\gamma;\alpha}=2H_{\alpha\beta}\eta_{\gamma}-H_{\beta\gamma}\eta_{\alpha}-H_{\alpha\gamma}\eta_{\beta}\,. \tag{3.4}$$

Multiplying (3.4) by η^{γ} and summing for γ, by virtue of (3.3) it follows that

$$\frac{1}{2}(\varphi_{\alpha\gamma;\beta}+\varphi_{\beta\gamma;\alpha})\eta^{\gamma}=H_{\alpha\beta}-\rho\eta_{\alpha}\eta_{\beta}\,,$$

from which by virtue of (1.3) and (1.6)

$$H_{\alpha\beta}=\rho\eta_{\alpha}\eta_{\beta}+\frac{1}{2}(\varphi^{\gamma}{}_{\alpha}\eta_{\gamma;\beta}+\varphi^{\gamma}{}_{\beta}\eta_{\gamma;\alpha})\,. \tag{3.5}$$

Solving (3.2) for $\eta_{\alpha;\beta}$ and substituting from the solution in the right hand member of (3.5), we obtain (3.1). Then we have the following theorem:

Theorem 3.1. *The second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in a K-space is of the form*

$$H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_{\alpha}\eta_{\beta}\,,$$

if and only if $\varphi_{\alpha\beta}$ satisfies the relation

$$\eta_{\alpha;\beta}=\lambda\varphi_{\alpha\beta}+A_{\epsilon\delta\beta}\eta^{\epsilon}\varphi^{\delta}{}_{\alpha}\,.$$

In particular, we have the following lemma in a constant curvature K-space:

Lemma 3.2. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in a K-space with constant curvature has the form*

$$H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_{\alpha}\eta_{\beta}\,,$$

then

$$\lambda\cdot\mu=0\,.$$

Proof. Since M^{2n+2} be a K-space with constant curvature, from (2.3) and (3.1), we get

$$(\lambda_{;\gamma})g_{\alpha\beta}-(\lambda_{;\alpha})\,g_{\beta\gamma}+(\mu_{;\gamma})\eta_{\alpha}\eta_{\beta}-(\mu_{;\alpha})\eta_{\beta}\eta_{\gamma}$$
$$+\mu(\eta_{\alpha;\gamma}\eta_{\beta}+\eta_{\beta;\gamma}\eta_{\alpha}-\eta_{\gamma;\alpha}\eta_{\beta}-\eta_{\beta;\alpha}\eta_{\gamma})=0\,.$$

Multiplying this equation by $\varphi^{\alpha\gamma}\eta^{\beta}$ and summing for all indices, by virtue of (1.3) it follows that

$$\mu\cdot\varphi^{\alpha\gamma}(\eta_{\alpha;\gamma}-\eta_{\gamma;\alpha})=0\,. \tag{3.6}$$

Substituting (3.2) into (3.6), we have the conclusion.

Theorem 3.3 *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in a K-space with constant curvature has the form*

$$H_{\alpha\beta} = \lambda g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta ,$$

then the hypersurface must be reduced to one of the following three cases:

(1) $H_{\alpha\beta} = \mu\eta_\alpha\eta_\beta$ *for a suitable function* μ, *i.e. rank* $(H_{\alpha\beta}) = 1$;

(2) $H_{\alpha\beta} = \lambda g_{\alpha\beta}$ *and* λ *is constant, i.e. the hypersurface is umbilical*;

(3) $H_{\alpha\beta} = 0$, *i.e. the hypersurface is totally geodesic.*

Proof. From the result of Lemma 3.2, we must have one of the following three cases; $\lambda=0$, $\mu=0$ and $\lambda=\mu=0$.
If $\lambda=0$, then we have the case (1).
If $\mu=0$, then the hypersurface is umbilical. Since in a hypersurface of constant curvature space the relation $H_{\alpha\beta;\gamma} - H_{\gamma\beta;\alpha} = 0$ is satisfied, from this relation and $H_{\alpha\beta} = \lambda g_{\alpha\beta}$, we see that λ is constant. Then we have the case (2).
If $\lambda=\mu=0$, then $H_{\alpha\beta}=0$, i.e. the hypersurface is totally geodesic.

When M^{2n+2} be a space of constant curvature, from the Gauss equation [2]:

$$K_{\alpha\beta\gamma\delta} = H_{\alpha\gamma}H_{\beta\delta} - H_{\alpha\delta}H_{\beta\gamma} + K_{ijkl}X^i_\alpha X^j_\beta X^k_\gamma X^l_\delta ,$$

we can easily obtain that the hypersurface is of constant curvature for above three cases. Thus we have the following corollary:

Corollary 3.4. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in a K-space with constant curvature has the form*

$$H_{\alpha\beta} = \lambda g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta ,$$

then the hypersurface is also of constant curvature.

Next, we assume that M^{2n+2} be an Einstein K-space, then we have the following lemma:

Lemma 3.5. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in an Einstein K-space has the form*

$$H_{\alpha\beta} = \lambda g_{\alpha\beta} + \mu\eta_\alpha\eta_\beta ,$$

then

$$\lambda(2n\lambda + \mu)_{;\gamma}\eta^\gamma = 0 . \tag{3.7}$$

Proof. Substituting (3.1) into (2.6) and multiplying by $g^{\alpha\beta}$ and summing for α and β, we get

$$2n(\lambda_{;\gamma}) + (\mu_{;\gamma}) + (\mu_{;\alpha})\eta^\alpha\eta_\gamma + \mu(2\eta_{\alpha;\gamma}\eta^\alpha - \eta_{\gamma;\alpha}\eta^\alpha - \eta_\gamma g^{\alpha\beta}\eta_{\beta;\alpha}) = K_{il}X^i_\gamma X^l .$$

Since M^{2n+2} is an Einstein K-space, we obtain by means of (3.2)

$$(2n\lambda+\mu)_{;\gamma}+(\mu_{;\alpha})\eta^{\alpha}\eta_{\gamma}-\mu\eta_{\gamma}A_{\epsilon\delta\alpha}\eta^{\epsilon}\varphi^{\delta\alpha}=0\ .$$

Multiplying the above equation by $\varphi^{\gamma\beta}\varphi_{\beta\kappa}$ and summing for γ, it follows that

$$(3.8)\qquad (2n\lambda+\mu)_{;\kappa}=(2n\lambda+\mu)_{;\gamma}\eta^{\gamma}\eta_{\kappa}\ .$$

Differentiating (3.8) covariantly with respect to u^{α} and multiplying by $\varphi^{\alpha\kappa}$ and summing for α and κ, we have the conclusion by means of (3.2).

Theorem 3.6. *If the second fundamental tensor $H_{\alpha\beta}$ of a hypersurface in an Einstein K-space has the form*

$$H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_{\alpha}\eta_{\beta}\ ,$$

then the hypersurface must be reduced to one of the following four cases:

(1) $H_{\alpha\beta}=\lambda g_{\alpha\beta}+\mu\eta_{\alpha}\eta_{\beta}$, *where* $2n\lambda+\mu=$ *constant;*
(2) $H_{\alpha\beta}=\lambda g_{\alpha\beta}$ *and* λ *is constant, i.e. the hypersurface is umbilical;*
(3) $H_{\alpha\beta}=\mu\eta_{\alpha}\eta_{\beta}$ *for a suitable function* μ, *i.e. rank* $(H_{\alpha\beta})=1$;
(4) $H_{\alpha\beta}=0$, *i.e. the hypersurface is totally geodesic.*

Proof. From the result of Lemma 3.5, if $\lambda\neq 0$, then $(2n\lambda+\mu)_{;\gamma}\eta^{\gamma}=0$. From this equation and (3.8), it follows that $2n\lambda+\mu=$constant. This is the case (1). Specially, if $\lambda\neq 0$ and $\mu=0$, then $H_{\alpha\beta}=\lambda g_{\alpha\beta}$, where λ is constant. Hence we have the case (2).
From (3.7) if $\lambda=0$, then we have the case (3). Specially, if $\lambda=\mu=0$, then $H_{\alpha\beta}=0$, that is, the hypersurface is totally geodesic.

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Department of Mathematics,
Hokkaido University

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