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COMPLETENESS OF COPI'S METHOD OF DEDUCTION

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Massey has pointed out in [2] that it is an open question as to whether Copi's method of deduction for propositional logic (**CMD**), as described in Chapter Three of [1], is complete in the sense of being able to validate every argument which can be proved valid by the use of truth-tables. It is here shown that **CMD** is complete in this sense, for its completeness follows from Theorem I below and the deductive completeness of the logistic system **R.S.** of Chapter Seven of [1].

The following lemma is required for the proof of Theorem I:

There is a formal proof by **CMD** *of the validity of* $q \vee (p \cdot \sim p) \cdot r \therefore q$.

Proof of the lemma:[1]

1.	$q \vee (p \cdot \sim p) \cdot r$	$/ \therefore q$
2.	$(q \vee (p \cdot \sim p)) \cdot (q \vee r)$	1, Dist.
3.	$q \vee (p \cdot \sim p)$	2, Simp.
4.	$(q \vee p) \cdot (q \vee \sim p)$	3, Dist.
5.	$q \vee p$	4, Simp.
6.	$\sim\sim q \vee p$	5, D.N.
7.	$\sim q \supset p$	6, Impl.
8.	$(q \vee \sim p) \cdot (q \vee p)$	4, Comm.
9.	$q \vee \sim p$	8, Simp.
10.	$\sim\sim q \vee \sim p$	9, D.N.

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1. The elementary valid argument forms of **CMD** used in constructing this formal proof are referred to by their abbreviations given by Copi on pages 42-43 of [1]. Note that because of Comm. for both disjunction and conjunction, formal proofs of the validity of $((p \cdot \sim p) \cdot r) \vee q \therefore q$, $(r \cdot (p \cdot \sim p)) \vee q \therefore q$, and $q \vee (r \cdot (p \cdot \sim p)) \therefore q$ can also be given. Thus any reference to the formal proof given for this lemma should be taken as referring to any one of these four formal proofs.

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11. $\sim q \supset \sim p$	10, Impl.
12. $p \supset q$	11, Trans.
13. $\sim q \supset q$	7, 12, H.S.
14. $\sim\sim q \vee q$	13, Impl.
15. $q \vee q$	14, D.N.
16. q	15, Taut.

Theorem I: *Corresponding to every derived rule which can be demonstrated in* **R.S.** *there is an argument which can be proved valid by Copi's method of deduction.*

The proof of the theorem relies on Copi's rule of Indirect Proof (**I.P.**), found on page 55 of [1], which may be given as: A formal proof of a contradiction from, say, $P_1, \ldots, P_n, \sim Q$ is an indirect proof of the validity of the argument $P_1, \ldots, P_n \therefore Q$. Thus, since **I.P.** is part of **CMD**, in order to prove the theorem it is sufficient to show that there is a formal proof of a contradiction from $P_1, \ldots, P_n, \sim Q$ by **CMD**, where $P_1, \ldots, P_n \vdash Q$ is any derived rule demonstrated in **R.S.** Proof of the theorem:

Assume that there is a demonstration in **R.S.** of the derived rule

(1) $P_1, \ldots, P_n \vdash Q$

Then by the assumption and the deduction theorem for **R.S.**, $\vdash P \supset Q$ in **R.S.**, where P is the abbreviation of the conjunction of $P_1, \ldots, P_n$. That is, since **R.S.** is deductively complete, by the assumption $P \supset Q$ is a tautology.

Now by **CMD** construct a formal proof of the validity of

(2) $P_1, \ldots, P_n, \sim Q \therefore N$

where N is the disjunctive normal form of the *wff* $P \cdot \sim Q$. Such a proof is always constructable since $P \cdot \sim Q$ is derivable from the premisses of (2) by **CMD** and Copi incorporates into his elementary valid argument forms the equivalences necessary to derive the disjunctive normal form of any *wff* in a formal proof.

Notice that N will be a disjunction in which every disjunct contains a contradiction. This is so because $P \cdot \sim Q$ is truth-functionally equivalent to a contradiction: $P \cdot \sim Q$ is truth-functionally equivalent to $\sim(P \supset Q)$ and by the assumption $P \supset Q$ is a tautology. Hence, by repeated bodily insertions of proper variants[2] of the formal proof given above for the lemma, the formal proof of the validity of (2) can be extended to a formal proof of some single disjunct of N from the premisses of (2). If this disjunct is not itself a contradiction it is a conjunction of a contradiction and some *wff*. Hence, by Comm. and Simp., the formal proof of the validity of (2) can be

2. A variant of a formal proof of the validity of an argument is to be understood as a formal proof of the validity of some substitution instance of that argument.

extended to a formal proof of some contradiction from the premisses of (2). That is, there is a formal proof of a contradiction from the premisses $P_1, \ldots, P_n, \sim Q$.

It should be noted that Theorem I does not show that there is a *formal proof* by **CMD** of arguments corresponding to derived rules which have been demonstrated in **R.S.** (as Copi claims there is on page 236 of [1]), but only that such arguments can be *proved* valid by **CMD**.

BIBLIOGRAPHY

[1] Copi, I. M., *Symbolic Logic*, The MacMillan Company, New York, 1954.

[2] Massey, G., Note on Copi's system. *Notre Dame Journal of Formal Logic*, v. IV (1963), pp. 140-141.

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