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NOTE ON COPI'S SYSTEM

GERALD J. MASSEY

Chapter Seven of reference [1] contains a fallacious proof that Copi's method of deduction for propositional logic (CMD) is complete in the sense of being able to validate every argument which can be proved to be valid by the use of truth-tables. CMD is set forth in Chapter Three of [1].

Copi tries to show that, corresponding to every proof from hypotheses in a certain logistic system, R.S., there is a formal proof or deduction in CMD. (See especially p. 236 of [1]). In the proof from hypotheses every line which is an hypothesis is justified as a premiss of the corresponding deduction. Since modus ponens is a rule of inference of both R.S. and CMD, an application of modus ponens in the proof from hypotheses is also a legitimate application of modus ponens in the corresponding deduction. Finally, according to Copi, corresponding to every line of the proof from hypotheses that is an axiom "we have in the corresponding formal proof of validity the Conditional Proof of that axiom." (p. 236). But this last assertion is ambiguous. It might mean (a) that the Conditional Proof (C.P.) of the axiom is bodily inserted into the deduction. But, since the C.P. of an axiom contains no line which is the axiom itself, the axiom would not necessarily appear in the deduction. Hence one could not be sure that the deduction always contained legitimate applications of modus ponens corresponding to those made in the proof from hypotheses. Furthermore, since the C.P. of an axiom begins by taking the antecedent thereof as a new premiss, the deduction would not validate the argument validated by the proof from hypotheses but rather a similar argument containing additional premisses. Or the assertion might mean (b) that, since we possess C.P.'s of the axioms of R.S., we are justified in writing them as lines anywhere in a deduction. But the rule of C.P. of Chapter Three, unlike the "strengthened" rule of C.P. of Chapter Four, authorizes no such procedure. Under the "weak" rule of C.P., this maneuver is tantamount to taking each axiom so introduced as a new premiss of the deduction. Thus the remark made in (a) about additional premisses applies here too.

If, however, the weak rule of C.P. is replaced by the strengthened rule,

CMD as thus augmented is indeed proved complete by Copi's argumentation. But whether CMD is complete without such augmentation remains an open question.

REFERENCES

[1] I. M. Copi. Symbolic Logic. New York, 1954 (Sixth printing, 1961).

Princeton University Princeton, New Jersey