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In Memoriam

CLARENCE IRVING LEWIS (1883-1964)

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Clarence Irving Lewis, philosopher and logician, was born in Stoneham, Massachusetts on April 12, 1883.¹ His father, a shoemaker, was blacklisted for trade-union activity. Young Lewis worked on farms and in shoe-factories till he entered Harvard College in 1902; he worked his way through college in three years. At about the age of thirteen, Lewis had fallen into philosophic wonderment without instruction, and discovered for himself the flux of Heraclitus (without the Logos), and the paradoxes of Zeno (without mathematical conceptions of infinity). Two years later, he began to read the Greek philosophers who had anticipated him, and Spencer's *First Principles*.

In college, Lewis divided his time between English, economics, and philosophy. In his final year, he took a course in metaphysics which William James and Josiah Royce divided between them. Lewis appreciated James's "swift way of being right", but Royce, with his "ponderous cogency", became and remained his ideal of a philosopher. After graduation, Lewis taught English in high school one miserable year; and spent two happy years at Boulder, Colorado, teaching English, assisting in philosophy, and beginning married life with his high-school sweetheart, Mabel Graves. He returned to Harvard for graduate work in 1908. He studied Plato with Santayana; and with lasting effect, Kant with Ralph Barton Perry (who required weekly summaries of the reading). He took metaphysics with Royce, and Perry's seminar in epistemology. With fellow-students and in his own mind, he fought out the battle between Perry's New Realism and Royce's absolute idealism. He rejected idealistic metaphysics, but felt the new realists and pragmatists did not do justice to the rationalistic arguments of the absolute idealists. He thought James and Royce had more in common than they recognized; he believed the "conceptualistic pragmatism" of his later Mind and the World-Order developed out of the same roots as Royce's "absolute pragmatism". Royce commented on Lewis's doctoral dissertation, "The Place of Intuition in Knowledge" (1910): "I thought you were principally influenced by Perry, but I find he thinks you are principally influenced by me. Between us, we agreed that perhaps this is original." Lewis's strong interest in logic developed through taking Royce's course in symbolic logic, assisting in this course (1910-11), and studying Volume I of *Principia Mathematica* which Royce put in his hands.

G. H. Palmer placed him as instructor at the University of California beginning 1911. Here he began publishing papers, mostly logical, became assistant professor in 1914, and wrote his *Survey of Symbolic Logic* (1918), the first substantial textbook of the subject in English. Except for an interlude of Army service during World War I, he remained at Berkeley till called back to Harvard as a Lecturer in 1920. He was made assistant professor the next year, associate professor in 1924, professor in 1930, and succeeded Perry as Edgar Pierce professor in 1946, holding this chair till 1953.

During the first two years of his teaching at Harvard, Lewis's study was the room containing the disorderly collection of Peirce papers. Browsing among these, he received many suggestions from a mind more congenial to his than was James's. But he preferred to develop his own thought rather than tackle the large job of editing the Peirce papers. His publications began to emphasize the epistemological interest which had preceded the logical while the latter continued. Mind and the World-Order (1929) and An Analysis of Knowledge and Valuation (1946) established Lewis as epistemologist of the first order. The logical and epistemological parts of An Analysis-some 360 pages-he regarded as necessary preliminary for the discussion of valuation. Meanwhile, Symbolic Logic (1932), with C. H. Langford as co-author, consolidated his position as the principal founder of the modern symbolic form of modal logic. While on the Harvard staff, Lewis held the presidency of the Eastern Division, American Philosophical Association (1934). He gave the Howison lecture in Berkeley in 1926 and the Carus lectures there in 1945. He received the Doctorate of Humane Letters from the University of Chicago in 1941 and the Butler gold medal in 1950 from Columbia University.

Lewis retired from Harvard in 1953. After teaching one year at Princeton, he went to live at Menlo Park, California, teaching intermittently at Stanford, with interim appointments at Michigan State and University of Southern California. He gave the Woodbridge lectures at Columbia in 1954, the Powell lectures at Indiana University in 1956, and the Honors College lectures at Wesleyan University in 1959. He received a \$10,000 prize for scholarship from the American Council of Learned Societies in 1961. He died on February 3, 1964 in Menlo Park of a heart defect of long standing. In his last decade, he concentrated on ethical questions. His main publications in this area (apart from An Analysis) were two short books, The Ground and Nature of the Right (1955) and Our Social Inheritance (1957). Lewis was a splendid teacher, conscientious, serious, always seeking the truth and seeking to convey it; never condescending and never pretending; not showy but with a wry wit and a salty humour. Basically, he was a good teacher because he was a careful scholar and a clear thinker, honest with himself and with his students. Many a graduate student who came to Harvard's philosophy department because of the repute of the University, the department, or some more famous professor, found the most lasting impression was that made by Lewis. One detail will give a hint of his probity. Having assigned his own book as a text, he told our class that, if we would bring the receipt from the bookstore, he would refund the amount of his royalty.

Lewis made his mark first in logic. The problems that interested him most were problems of theory-philosophy of logic, we may say; but he never shirked the technical problems that are bound up with the theoretical. Lewis was much impressed by *Principia Mathematica* (**PM**). But he was troubled from the first by the paradoxes of material implication: e.g., ''A false proposition implies any proposition,'' ''A true proposition is implied by any.'' This kind of 'implication', he saw, could not be the basis of logical deduction. One cannot, for example, logically deduce every true proposition from any given proposition. The difficulty arose, as Lewis saw it, because the logic of propositions of **PM**-following the successful tradition of Boole-was truth-functional or *extensional*. Deductive inference, however, depends upon relations of meaning, and would therefore seem to require an *intensional* logic. But no adequate logic of intension had ever been developed.

Lewis found himself confronted by two sorts of problems. First, could he develop an exact logic of intension, analogous to the existing logic of extension? "And is the intensional analogue of material implication the relation upon which deductive inference is usually founded?" Second, if there are different possible exact logics, how can one determine their validity or truth, since "logic itself provides the criteria of validity used elsewhere, and the application of these to logic would be *petitio principii*?" [32]

Lewis starts in [2] (1912) from a contrast between the truth-functional disjunction of PM and an intensional disjunction (both non-exclusive). For example, "(1) Either Caesar died or the moon is made of green cheese, and (2) Either Matilda does not love me or I am beloved... The second disjunction is such that at least one of the disjoined propositions is "necessarily" true" (p. 523). Intensional disjunction could be related to intensional (or strict) implication by a definition analogous to PM's definition of material implication. Thus the intensional disjunctive of (2) above "is equivalent to 'Matilda loves me implies that I am beloved'."²

He considers which of the **PM** postulates fail for intensional disjunction, and finds an analogue lacking only for Addition: "if q, then either p or q" does not hold for intensional "or". At this time he accepts the intensional analogue of **PM**'s Association made by replacing *every* material disjunction by intensional disjunction: $[p \lor (q \lor r) \supset q \lor (p \lor r)]$. Such a formula for intensional disjunction is finally rejected in the *Survey*. It leads to consequences incompatible with the distinction of empirical and necessary truth.³ In his early papers, Lewis sometimes defines strict implication in terms of intensional disjunction, takes it as undefined once [5], and finally (1914) [9] defines it as the impossibility of the antecedent and not the consequent. The latter remains his definition henceforth.

An important stage was the exposition of the system of Strict Implication in the *Survey*. The primitive ideas were: propositions, p, q, r, etc.; negation, -p; impossibility, $\sim p$; the logical product, pq; and (strict) equivalence, p = q, "the defining relation". (The last is also defined.) Strict implication is defined: $p \exists q = \sim (p - q)$.

The eight postulates unfortunately included one mistaken one, repeated from a 1914 article [9]:

1.8 $p \rightarrow q = \sim q \rightarrow \sim p$

1.8 is equivalent to the pair:

2.2 $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p),$ 2.21 $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$

E. L. Post showed that with 2.21 one can prove -p equivalent to $\sim p$, destroying the distinction between material and strict implication. Lewis amended the system by replacing 1.8 by 2.2 as postulate [15]. This amended system-later called S3-was the first correct and reasonably complete system of strict implication ever given.⁴ Lewis noticed that his strict implication had some unexpected and 'peculiar' properties, analogous to the paradoxes of material implication. An *impossible* proposition strictly implies any proposition; a necessary proposition is strictly implied by any. Not one to evade a difficulty, he faced it frankly in the Survey and later writings. He found that, apart from the definition of strict implication, commonly accepted principles would lead to such paradoxical results. For example, "Today is Monday" implies "Tomorrow is Tuesday". Hence "Today is Monday and the moon is not made of green cheese" implies "Tomorrow is Tuesday". Then, by the principle of the antilogism, "Today is Monday but tomorrow is not Tuesday" implies "The moon is made of green cheese".

The Survey includes a detailed account of Lewis's research in the development of symbolic logic. He reports on the attempts of Leibniz, J. H. Lambert, and G. F. Castillon to develop an intensional logic of terms, (with translation of substantial fragments from Leibniz,) and G. J. von Holland's attempt at an extensional calculus. Lewis got no help for his intensional logic from these efforts. (He would have got more help from such medievals as Buridan, Albert of Saxony, and Paul of Venice.) He continues with the more successful extensional logic from De Morgan and Boole to **PM**. This is still a useful supplement to later accounts. An interesting feature of the Survey is his "heterodox" view of mathematics in the last chapter—an early statement of an extreme formalism.

After the Survey, various European and American logicians worked on

strict implication. By Bernays's method it was shown that S3 did not reduce to Material Implication. O. Becker proposed various formulas for implication or equivalence between modalities of different degrees. The way was prepared for Lewis's next step. In 1932, *Symbolic Logic* [33] appeared, with C. H. Langford as co-author; the parts concerned with the development and justification of strict implication were written by Lewis.

Lewis was doubtful whether some of the theorems that had been proved in S3 were valid principles of deduction. He therefore retreated to a weaker system, S2, with a stricter implication. It had not yet been proved that S2 lacked the doubtful theorems. Hence he formulated a still weaker system, S1, as a line of possible further retreat. In *Symbolic Logic*, the primitive ideas are as in the *Survey*, except that, instead of *impossibility*, he takes as primitive "Self-consistency or possibility: $\Diamond p$." A "wide meaning" of possibility is intended, "namely, logical conceivability or the absence of self-contradiction." This may be called *absolute* possibility, as contrasted with the relative possibility that means "consistent with the data."

The "operations" (transformation rules) are: Substitution (a) of logical equivalents, (b) for variables; Adjunction; and *modus ponens* for strict implication.

To this basis are added seven postulates for S1:

$$pq \exists qp,$$

$$pq \exists p,$$

$$p \exists pp,$$

$$(pq)r \exists p(qr),$$

$$p \exists -(-p),$$

$$p \exists q \cdot q \exists r : \exists \cdot p \exists r,$$

$$p \cdot p \exists q : \exists \cdot q \cdot$$

(McKinsey showed that the fifth was redundant.)

An eighth postulate is added to make S2:

B8 \Diamond (*pq*) \neg \Diamond *p*.

To exclude material implication as an interpretation of the hook, an existence postulate is included:

B9 $(\exists p,q): -(p \rightarrow q) \cdot -(p \rightarrow -q)$

Lewis identifies more inclusive systems which deserve consideration as possible systems of strict implication. The corrected *Survey* system is called S3. S3 is also given by adding 2.2 of the *Survey* system to S1.

Adding to S1 one of Becker's reduction formulas,

C10
$$-\diamondsuit -p \rightarrow -\diamondsuit -p$$
,

gives S4. Adding to S1 another Becker formula,

C11 $\Diamond p \rightarrow \neg \Diamond p$, gives S5, Becker's "six-modalities calculus".

Lewis also suggests a peculiar postulate,

C13 $\Diamond \Diamond p$,

which is inconsistent with C10, but could be added to S2 or S3 (giving systems later called S6 and S7 respectively).

The systems S1-S5 form a series with decreasing strictness of the implication, each a sub-system of those with higher numbers. After it was shown (1934) that S2 was distinct from S3, Lewis made definite his tentative assertion in *Symbolic Logic* that S2 is to be regarded as *the* System of Strict Implication, i.e., the one whose " $p \rightarrow q$ " best represents "q is deducible from p" as ordinarily used.

Symbolic Logic also considers other possible 'rivals' of Material Implication, namely, the truth-implications (relations satisfying certain formal conditions) of the many-valued logics of Łukasiewicz, especially his 3-valued system. Formulas of these truth-value systems are logical truths if and only if they are tautologies, i.e., they take the value *true* (or a 'designated value') for every substitution for the variables. And a truth-implication is sufficient justification for inference from its antecedent to its consequent just in case the relation holds tautologically. Just as p strictly implies q if it is tautological (or logically necessary) that $p \supset q$, so p strictly implies q if, for some truth-implication I, it is tautological that pIq. Hence valid inferences in truth-value systems are always based on strict implications, never on a *mere* truth-value relation.

With the completion of *Symbolic Logic*, Lewis felt he had essentially solved his problem of developing an exact logic of intension, whose implication is the usual basis of deduction. As for predicate calculus, apart from suggestions in the *Survey*, Langford's indications in chapter 9 of *Symbolic Logic*, and Lewis's calculus of predicative functions intended to serve as "convenient basis for an intensional logic of terms" [48], he was content to leave this to others such as Dr. Ruth B. Marcus. He would defend or explain strict implication as occasion arose [40, 44], and added a "Final Note on System S2" to the 2nd edition of *Symbolic Logic*, to make that "the permanent record of Strict Implication".

There remained his second basic problem: *since* there are different exact logics, is there an issue of truth to be decided between them? What criteria can one use without question-begging? His answer is in [32], chapter 8 of [33], and [34, 35, 36, 38]. Part of the answer lies in the fact that he was convinced—by metalogical considerations, we may say—that every law of each exact logic is *'absolutely true''*, being analytic or tautological. Each system is consistent and self-affirming, its principles implied by their own denial. But to use logic as a canon of inference, we must choose among the systems and use at most two or three; else we have *'*not a canon, but a chaos.'' The choice is among equally true systems, and so must be made by *'*criteria of convenience*''*, by *''pragmatic considerations such as simplicity or comprehensiveness or accord with our most frequent purposes of inference. On these grounds, there are perfectly definite . . . reasons for choosing the usual meaning of <i>'implies'''* [38]. Let

me comment that simplicity and comprehensiveness may point in opposite directions. The former may favor extensional logic, the latter an intensional logic—perhaps even richer than Strict Implication.

This problem in the philosophy of logic had taken him back into epistemology. In fact, it was largely the impact of grappling with such problems, on the background of his reaction to the conflict of Royce, Perry, and the pragmatists, that led him to the view he called "conceptualistic pragmatism". *Mind and the World-Order* [29] was the first full statement of this. He distinguished two elements in knowledge: the concept and the sensuously given. All *a priori* knowledge—contrary to Kant—is analytic of conceptual meanings. He took **PM** as showing the deductive development of pure mathematics from logical analysis of concepts. But the choice of concepts or systems for application to experience is pragmatic. Any judgment of empirical reality is essentially predictive, hence only probable. That experience is capable of conceptual interpretation requires no metaphysical assumption of the conformity of experience to categories; "it could not conceivably be otherwise." These are the basic theses of [29].

He thought to turn now to ethics, which he had always considered the most important branch of philosophy. However, several factors combined to make him devote nearly two thirds of his next book to logic and epistemology. One was the fact that criticisms of his book [29] were appearing; also other related but divergent books were appearing. Most important was the fact that valuation in his view was a species of empirical knowledge. To remove misunderstandings and to combat emotivist ethical theories, he found that a necessary preliminary was a more technical elaboration of his logical-epistemological ideas than he had previously made.

The main feature of Book I of *Analysis of Knowledge and Valuation* [44] is a careful account of the various modes of meaning of terms and propositions. Lewis has an unusual definition of "proposition": "a term capable of signifying a state of affairs". (A term is a linguistic expression, but its identity depends upon both its symbol and its meaning.) Assertion is extraneous to a proposition. The proposition (asserted e.g. in the statement "Mary is making pies") is expressed by a "that" clause ("that Mary is making pies now") or by a participial phrase ("Mary making pies now"). All true propositions denote the same thing, viz. the actual world; false propositions denote nothing.

"All truths of logic are analytic formal statements"; but "not all analytic statements belong to logic". For example, "All birds are bipeds" is analytic, but is not regarded as a statement of logic. But the boundary line can be drawn only by "some convention or pragmatic decision."

Book II analyzes empirical knowledge. There are three kinds of empirical statements. (1) What is presently given in experience is formulated in *expressive statements*. (2) There are *terminating judgments*, which predict possible experience. They are of the form "(S being given,) if A then E," where A is a mode of possible action, E an expected consequent in experience, and S the sensory cue. (3) Non-terminating

judgments assert objective reality. They are so named because no limited set of predictions can exhaust their significance. Lewis finds it necessary to give his own account of probability. Underlying both *a priori* and empirical theories he finds "the thesis that *a probability is a valid estimate of a frequency from the given data.*"

Book III develops a theory of valuation on the bases laid down. It is a naturalistic as well as a cognitive theory. It must start from value as given in immediate experience. The best name for this is *satisfaction*. This is intrinsic value. Extrinsic value attaches to objects as they conduce to intrinsic value. He rejects a Benthamite calculus, however, because of the *Gestalt* characteristics of experience. Although this work covers a number of important topics commonly handled in moral theory, it does not, on his view, get into the distinctive questions of ethics. For, he concludes: "Valuation is always a matter of empirical knowledge. But what is right and what is just, can never be determined by empirical facts alone."

He set forth a rationalistic ethics in two short books, *The Ground and Nature of the Right* [56] and *Our Social Inheritance* [58], and in a number of papers, mostly unpublished. Some of these, including four lectures on "Foundations of Ethics", are to be published by Stanford University Press.

Let us attempt to sum up the influence of Lewis's writing and teaching. He is the principal founder of the modern symbolic treatment of modal logic and theory of entailment. He contributed to the history of logic. His *Survey* was a pioneer textbook, and both of his logic books combined pedagogical value with important contributions to the subject. His contributions to philosophy of logic and to epistemology are important and influential. He has heightened the awareness of the interrelations between logic, epistemology, and value theory; and has made a strong case for cognitivism in valuation. It is too early to estimate his influence in ethics; here one must await the reaction to his posthumous papers.

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NOTES

1. Biographical information comes from the two autobiographical accounts by Lewis [32, 63], and from the excellent memorial by Professor Donald C. Williams in *Philosophy & Phenomenological Research*, 27 (1965), pp. 159-172. Dr. Williams' article should be read for a warm and well informed account of Lewis as a man.

The exposition of Lewis's thought is based mainly on my study of his writings, no doubt influenced by study under him more than thirty years ago. The principal secondary source is *The Philosophy of C. I. Lewis*, ed. by P. A. Schilpp (Open Court, 1968). The main bibliographical source is E. M. Adams' "The Writings of C. I. Lewis" in the Schilpp volume cited.

- 2. This quotation illustrates Lewis's occasional disregard—in the tradition of **PM**—of strict conventions for ordinary language. Critics like Quine would say that Lewis should write "implies" between the *names* of statements, not use it as a statement connective like "if . . . then". What Lewis's system requires, however, can be correctly expressed by forms such as "That Matilda loves me implies that I am beloved," or "Necessarily, if Matilda loves me then I am beloved."
- 3. We note that the mistake regarding association for intensional disjunction related two expressions each of the second degree of modality (i.e., with one modal expression within the scope of another.) And the mistaken transposition of 1.8 (below) involved the relation of an expression of the second degree to one of the first degree. Logical intuition most easily misjudges the relation of modal expressions when one or both is of degree higher than one.

It seems, by the way, that Professor Milton Fisk is repeating Lewis's early mistake in assuming S21, an associative law for intensional disjunction, in A *Modern Formal Logic* (1964), p. 38. This leads to a triadic 'commutation' for the intensional conditional (p. 47) which has undesirable consequences.

4. I count as correct and 'reasonably complete' also the systems S1 and S2 of [33], and S0.5 of E. J. Lemmon (*Journal of Symbolic Logic*, 1957); for they all contain as theses all valid formulas of the first degree of modality. None of them is complete in the sense I deem most appropriate: that if P yields Q as a theorem, then there is a theorem that $P \neg Q$. The *Survey* contains a fragmentary system, "the Calculus of Consistencies", with only correct assumptions; but this lacks conjunction and all binary truth-functions.

Hugh MacColl (whom current historians usually rob of his "a") had many correct formulas, and perhaps only correct ones, in his *Symbolic Logic and its Applications* (1906); but he does not exactly give a formal deductive system.

5. This bibliography is adapted from that of Professor E. M. Adams, which appears in *The Philosophy of C. I. Lewis*; he also made it available to me in two earlier mimeographed forms. Dr. Adams' bibliography includes also the unpublished papers of Lewis, and many details about the writings which I have eliminated. I have made some minor additions and changes.

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