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THE RULE OF PEIRCE

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Wajsberg's [1] gave several bases for classical implication in which *CCCpqpp* (Peirce) was replaced by the rule: *From* $CC\alpha\beta\alpha$ *to infer* α. Our question is whether this can be done with the Łukasiewicz base which generalizes so many of Wajsberg's other results, *CCpqCCqrCpr*, $Cp C\alpha\beta$, *CCCpqpp*, (the rules of detachment and substitution always here being taken for granted). The answer is negative, by the matrix

C	*1*	*2*	*0*
**1*	*1*	*0*	*0*
2	*1*	*1*	*0*
0	*1*	*1*	*1*

which is part of Wajsberg's Matrix A. This verifies Syllogism and *CpCqq*, the rules of detachment and Peirce are satisfied, but $CpCCppp = 0$ when $p = 2$. However Peirce can be replaced in the Łukasiewicz base by the modified rule:

From $C\gamma CC\alpha\beta\alpha$ *to infer* $C\gamma\alpha$,

Since this rule with syllogism yields Peirce. In fact, let $\mathbf{P}.n$ be the most general result of applying the modified rule to thesis n, and $\mathbf{D}.m.n$ be the most general result of detaching a substitution in thesis n from a substitution in thesis m, and let *CCpqCCqrCpr* be thesis 1; then Peirce = **P.D.1.P.1.**

REFERENCE

[1] Wajsberg, M., Metalogische Beiträge, *Wiadomości Matematyczne* 43 (1937), pp. 1-38; Metalogische Beiträge II, ibid. 47 (1939), pp. 119-139. Both papers are translated in *Polish Logic 1920-1939*, ed. Storrs McCall (Oxford, 1967).

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