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SCHEMATIZING DE MORGAN'S ARGUMENT

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A century and a quarter ago Augustus De Morgan challenged anyone to deduce syllogistically from 'Every horse is an animal' that 'Every head of a horse is the head of an animal.'[1] His challenge went unanswered, and history gives this argument the credit of being the first to decisively show the shortcomings of Aristotelian logic. Modern logic, encompassing relative terms, can show the formal validity of the argument and is thus rightly thought a great advance. Given the venerability of the argument it is surprising that to the present day it is almost universally schematized incorrectly.

The premise causes no problem, being schematized as

$$(1) \qquad (x)(Fx \supset Gx)$$

with appropriate understanding of 'F' and 'G'. It is the conclusion of the argument which is invariably gotten wrong. Look in standard introductory logic texts such as those by Quine, Kalish, and Montague, Copi or Suppes and you will find that this sentence (or its equivalent) is schematized as

$$(2) \qquad (y)[(\exists x)(Fx \,.\, Hyx) \supset (\exists x)(Gx \,.\, Hyx)]$$

where 'Hyx' is read 'y is a head of x'. I think there are reasons showing this is a mistake; further, there are reasons showing why the mistake is not usually noticed.

The correct schematization of the conclusion is

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1. I have not found exactly this argument in De Morgan. The argument occurs with 'horse' replaced by 'man', *cf. Formal Logic* (1847), p. 114; *On the Syllogism and Other Logical Writings*, ed. by Peter Heath (Routledge & Kegan Paul, 1966), pp. 29 and 216.

   Richard Schubert has pointed out to me that the example with 'horse' occurs in *Principia Mathematica* *37.62 which in turn refers to W. S. Jevons, *Principles of Science* (1887), p. 18. Jevons says De Morgan used the example in conversation.

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(3) $(x)(y)(Fx \,.\, Hyx \,.\supset.\, Gx \,.\, Hyx)$

where the predicate letters are read as before. Schemata (2) and (3) are not equivalent; (3) implies (2) but not vice versa. To see this, interpret 'Fx' as 'x is female', 'Gx' as 'x is male', and 'Hyx' as 'y is a child of x', and let the universe of discourse be humans. On this interpretation (2) expresses the truth that 'Everybody who has a mother has a father.' (Children of widows are handled by taking 'is a child of' tenselessly. A more formal interpretation would avoid these complications, but my purpose is explanatory; the astute reader can think up his own examples.) But (3) would express the falsehood, 'Everybody's mother is his father.'

One way of explaining why (3) is false and (2) true on the latest interpretation is to point out that (3) says that the female whom y is a child of *just is* the male y is a child of, while (2) in no way suggests any such identification. It is precisely this difference which shows (2) to be an inadequate schematization of De Morgan's conclusion; for (2) leaves open the possibility that the animal which y is head of may be unrelated to the horse which y is head of. Surely this is too weak an understanding of the conclusion. The strengthening required is indicated in (3), which says that whatever horse y is a head of *is* an animal y is a head of. I take this to be the obvious intent of De Morgan's conclusion.

Another bit of evidence of (2)'s inadequacy is the fact that it can be derived from a much weaker premise than the one De Morgan gives. For example,

(4) $(y)\,[(\exists x)Hyx \,.\supset.\, (\exists x)(Fx \,.\, Hyx) \,.\, (\exists x)(Gx \,.\, Hyx)]$,

ugly as it is, implies (2). For the sense of it, notice that on our second interpretation it expresses the truth that whoever has a parent has both a mother and father, while on De Morgan's interpretation it expresses the falsehood that every head is the head of a horse and the head of an animal. It does not imply (3), which I take to be another mark in (3)'s favor.

Why then is (2) so commonly given as the schematization of De Morgan's conclusion? Why is the difference between (2) and (3) not noted? The reason is that the schematization always occurs in a context where (1) is being assumed as a premise. But when (1) is true you will never have the case that (2) is true and (3) is false, since (1) implies both (2) and (3). In other words, given that (1) is true, there is no difference between (2) and (3) as regards truth and falsity; and since everyone who schematizes the sentence does so where it is the conclusion of an argument which assumes (1) is true, one can account for their failing to notice the difference between (2) and (3). But being able to explain the practice does not justify it. It is time the practice was changed.

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