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THE THEORY OF CONSEQUENCE IN THE LATE FIFTEENTH AND EARLY SIXTEENTH CENTURIES

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Part One. In this paper I intend to examine the treatment accorded to consequences by a group of writers from the late fifteenth and early sixteenth centuries, although I shall make some reference to earlier periods. The subject of consequences (or valid inference) is of central importance to the historian of logic because those who discussed it covered such a wide range of logical issues, including criteria for validity, problems of self-reference, the status of the so-called paradoxes of strict implication, and the systematization of valid inference forms. Indeed, a large part of semantics and the whole of formal logic could be subsumed under this general heading. Whether the authors themselves fully appreciated that this was so is unfortunately not such an easy question to answer, for those I am concerned with frequently leave the reader in doubt as to their view of the relation of consequences to the rest of logic. So far as they discussed the matter, syllogistic was seen to be consequential in nature,[1] but they certainly did not make the subordinate position of the syllogism as clear as Burleigh had in the fourteenth century, or indeed as Andreas Kesler was to do in the seventeenth century.[2] A good guide to the way they viewed the problem is to see where consequences were discussed. A very few authors, including J. Major, A. Coronel and J. Almain, devoted a whole treatise to them, but generally speaking they came in on the coat-tails of other topics so far as separate treatises were concerned. They appear at the beginning of Dolz's treatise on the syllogism, at the end of Celaya's treatise on supposition and under 'hypothetical propositions' in the treatises on opposition written by R. Caubraith and F. Enzinas. The best places to look for a discussion of consequence turn out to be commentaries on Peter of Spain, where they appear either as an appendage to the *Parva Logicalia* or under the heading of 'hypothetical propositions', and, of course, general textbooks of logic. In these, a separate tract was sometimes devoted to consequences, as it was by C. Javellus, but more usually they were associated with the syllogism, whether as an introduction to it or, sometimes, as an appendix to it. Savonarola, for instance, said all

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he had to say of consequences in a section on the powers of the syllogism. The bibliography at the end of this paper should give a fairly clear picture of the situation; though it must be noted that the majority of commentaries and textbooks belonging to the sixteenth century did not mention consequences at all.

Another aspect of consequences about which our authors tend to be vague is the specification of what a consequence is in what they called the material as opposed to the formal sense. Menghus Blanchellus Faventinus listed four possible formal definitions with which all logicians would have been acquainted, namely that a consequence is a relation of inference between a consequent and an antecedent, that it is an intellectual act, that it is a passion or something brought about by the act of inferring, and that it is a proposition inferring consequent from antecedent; but all he said of the material sense was that a consequence is an aggregate of antecedent and consequent together with an inferential sign.[3] The most usual form of words was "a consequence is an *oratio* with an antecedent, a consequent and a *nota illationis*" such as 'si', 'quia', 'ergo', 'ideo', or 'igitur'.[4] The presence of 'si' and the use of words like 'antecedent' and 'consequent' rather than 'premiss' and 'conclusion' suggests that the distinction between a consequence and a conditional proposition was blurred; but although conditionals frequently appeared as instances of consequences, it was made quite clear in practice that a consequence was not a proposition but a sequence of propositions which purported to be an instance of a rule justifying that sequence. In accordance with this, while propositions were said to be true or false, consequences were said to be valid or invalid. To be more precise, they could be assessed in two ways, as instances of rules, or as instances of valid rules. They could fail because the rule appealed to did not apply, as was often said to happen in the case of consequences involving self-referential propositions, or they could fail because the rule itself failed. While consequences were clearly seen to be distinct from conditional propositions, the relationship between the two was rightly regarded as close, especially as conditionals were interpreted as exemplifying strict rather than material implication.[5] Clichtoveus remarked that every good consequence was a true conditional and vice versa;[6] and the Cologne commentators said with regard to syllogistic consequences that an 'antecedent' could be a conjunction of propositions rather than a single proposition, and that if such an antecedent were taken together with the consequent, it would make a conditional proposition.[7]

Some of the uncertainty attached to the definition of a consequence was reflected in the debate as to whether a consequence could be invalid or not. If a consequence was viewed merely as an *oratio* the question did not arise, but if, as was sometimes the case, the formal condition that the consequent should be derived from or follow from the antecedent was added, it seemed contradictory to turn around and deny that such a relationship held. One way of dealing with the problem was to say that 'consequence' could be taken in a broad as well as a strict sense;[8] another was to say that the division of consequence into valid and invalid was the division of a genus

into its species; and the third involved the claim that an invalid consequence stands to a valid consequence as a painted man stands to a living man.[10] The relationship is one of analogy only. Celaya alone realized that a third division could be made when he said that one could have not only invalid consequences, but formally invalid, or contravalid, ones as well.[11] He cited '$\ulcorner P \vee \text{-}P \urcorner \rightarrow \ulcorner Q \,.\, \text{-}Q \urcorner$' as an instance of what he had in mind.

The topic to which our authors rightly devoted most of their time and ingenuity was that of the definition of a valid consequence, but before I examine their arguments it is necessary to make some preliminary remarks about their attitude to propositions and about the place semantical issues played in the general discussion. A proposition does not seem to have been viewed as some kind of timeless entity, an intermediary between a sentence and a state of affairs, but rather as an actual occurrent, that is, as a declarative sentence which was either written, spoken, or thought. That this was so, complicated matters in two ways. In the first place, it was possible to conceive of a consequence which failed because of the non-existence of one of its parts. In the second place, some propositions were such that their very existence produced a paradoxical situation. "No proposition is negative" was a favourite example of this. Nor could the authors ignore such awkward examples on the ground that the paradox was not a formal one, since the distinction between formal and material consequences, or between syntactical and semantical issues, was a subordinate one. Any definition of validity produced was supposed to cover both inferences like '$\ulcorner P \,.\, Q \urcorner \rightarrow P$' and inferences like '"Smith is a bachelor" implies "Smith is male."' They did not seek to escape from problems of self-reference or problems produced by the reinterpretation of constituent parts (suppose "bachelor" came to mean "has red hair"?) by retreating into a formal system containing either uninterpreted theorems, or theorems interpreted in the austerest possible manner, by the assignment of letters, numbers, or members of a domain to the constituent parts.

The starting point for the discussion of validity was an examination of the most obvious definition, "A consequence is valid if and only if it is impossible for the antecedent to be true and the consequent false." A few people accepted it as adequate,[12] but many felt that it expressed neither the necessary nor the sufficient conditions for validity. There were two reasons for rejecting it as a necessary condition. In the first place, it was sometimes claimed that where self-reference was involved, one could have a valid consequence with a true antecedent and a false consequent, or at least, one which was not true. Pardo, for instance, accepted "Every proposition is affirmative, therefore no proposition is negative" as valid, even though the consequent could not be true. To be true it must exist, and the moment it comes into existence it falsifies itself.[13] In the second place, such a definition overlooks the requirement of existence. Almain felt that "Socrates runs, therefore a man runs" could have a true antecedent and a consequent which was not true because it did not exist. As a result, his first step towards an adequate definition was to say "It is impossible for the antecedent to be true when an existing consequent is false."[14] However,

as Pardo's example suggests, there were situations in which the actual existence of a proposition was most inconvenient, and authors like Caubraith modified their final definition in such a way that existence was not viewed as a necessary condition.

The fullest and most elaborate discussion of the question whether the definition provides a necessary condition for validity was offered by Niphus, though he obviously owed much of his argument to Pseudo-Scotus.[15] Using the same example as Pardo, he said that the definition does not cover the obviously valid consequence "Every proposition is affirmative, therefore no proposition is negative." Here the antecedent can be true, because all negative propositions could be annihilated, yet in such a situation the consequent would be false, indeed impossible, because it would not exist. If it did exist, then one could immediately demonstrate the truth of its contradictory, "Some proposition is negative." One could try to modify the definition by introducing the requirement that the antecedent and consequent should be formed together, but this will not work, for if all negative propositions are annihilated, the consequent cannot be formed at all. Hence the definition must be modified by the addition of the words "if they were formed together,"[16] so that a counter-factual conditional is produced. This is all right, he said, because if it is the case that all propositions are affirmative, it certainly follows that no proposition is negative, even though "No proposition is negative" is itself impossible.

The chief quarrel with the definition in terms of truth was that it failed to give a sufficient condition for validity, allowing as it did a number of obviously unacceptable consequences.[17] The two fullest discussions are to be found in the works of Celaya and Almain, so I shall concentrate upon their arguments.[18] Celaya took as his chief example "No proposition is negative, therefore man is an ass." One can argue, he said, that it is impossible to have a true antecedent and a false consequent here because if the antecedent is true then it is, as an existent proposition false, and if the consequent is false then the falsity of the antecedent follows by an acceptable rule. Yet the consequence is clearly invalid, because it violates the rule that an impossible proposition cannot follow from a possible proposition. Those who attempt to solve the problem by saying that the antecedent refers to a future time and hence can be true now, while the consequent is false, are easily refuted by the formulation of a new antecedent, "No proposition is or will be or was negative." Almain's example was virtually the same, namely "No proposition is negative, therefore God does not exist", which is, he said, invalid because it violates the rule that in a valid consequence the contradictory opposite of the consequent implies the contradicatory opposite of the antecedent. He went on to discuss the whole matter of truth and possibility. It does not follow, he said, that because a proposition cannot be true, it is impossible, or that because it cannot be false, it is necessary. One can have two propositions which are compossible, like "No proposition is negative" and "God exists", even though they cannot be true together; and one can have two propositions which are synonymous, although only one is true, as where "There is no spoken

proposition" is both spoken and written. Finally, a proposition like "Some proposition is negative" can be false, even though its contradictory, "No proposition is negative," cannot be true. In other words, a proposition is possible when the state of affairs it refers to is possible, but it is possibly true only when its existence does not conflict with the state of affairs described.[19] Almain, like Celaya and a number of others, turned therefore towards a definition of validity in which the notion of possibility was clearly applied to states of affairs rather than to the truth of propositions.

The new definition offered by Celaya took the form, not so much of a new set of words, as of a new gloss upon the old set of words. He still held that a consequence was valid if and only if it was impossible for the antecedent to be true and the consequent false, but he drew a distinction between two senses of 'true', and the two corresponding senses of 'false'.[20] The antecedent could be true in the sense that it signified things to be as they in fact were, or it could be true in the sense that it signified things to be as they in fact were by means of a true proposition. Only the first sense is applicable here. Almain, however, made his meaning immediately explicit when he dropped all reference to truth and said that in a valid consequence it was impossible for whatever the antecedent signified to obtain without what the consequent signified also obtaining.[21] This definition was not, of course, original to Almain, for it goes back to Buridan and Albert of Saxony;[22] and some of Almain's contemporaries like Caubraith gave very elaborate versions of it in order to guard against the pitfalls which are contained even in this new formulation.[23]

In the first place, existence was still seen to be a problem. Caubraith took as his example "God exists, therefore some proposition is indefinite." He did not wish to accept such a consequence as valid, yet if one admits that things are as signified by the antecedent, one is committed to the claim that the antecedent proposition exists, which in turn renders the conclusion true, so that it is impossible for things to be as signified by the antecedent without their being as signified by the consequent.[24] His way of escape, which was also used by men like Major, Pardo, Enzinas and Soto, was to add the clause "or as can be signified."[25] One can agree that things are as *can* be signified by "God exists" without committing oneself to the existence of any proposition. Other awkward cases, like "No proposition is negative, therefore some proposition is negative" or "Every proposition is particular, therefore some proposition is universal" can equally well be ruled out;[26] whereas legitimate cases like "Every proposition is affirmative, therefore no proposition is negative" are not affected.

The problems arising from syntactical self-reference were easily dealt with, but those arising from semantical self-reference were more intractable. Enzinas, Caubraith and Celaya all produced the same two examples of this second kind of self-reference: "This consequent does not signify things to be as they are, therefore this consequent does not signify things to be as they are" and "This consequence is valid, therefore man is an ass."[27] The first case seemed to be an instance of the valid rule leading from synonym to synonym, yet when the consequent referred to is its own,

the antecedent signifies (or can signify) things to be as they are, but the consequent does not. The second case seems to be invalid, for the impossible is derived from the possible, yet it is not possible for the antecedent to signify things to be as they are without the consequent also so signifying, for it is not possible for the antecedent to signify things to be as they are at all. Both Enzinas and Caubraith added two clauses to their definition, that the consequent should not have "does not signify things to be as they are" appended to it, and that the entire consequence should not invalidate itself. Celaya said that his definition did not apply to insolubles (as semantical paradoxes were called), where it is quite possible for an invalid consequence to be subordinated to a valid one, as in the first case above. He added that in the case of insolubles one could have a true conditional which was an invalid consequence, as in "If this is a valid consequence, then man is a lion", or a valid consequence which was a false conditional, as in "If this conditional is true, man is an ass." This is valid, because the antecedent is impossible, but a false conditional because it falsifies itself. More accurately, one might say that he is denying the equivalence between '$P \rightarrow Q$' and "'$P \nrightarrow Q$' is valid" which was normally assumed to hold. For further details he, like Enzinas, referred the reader to his treatise on insolubles.

Since propositions were regarded as occurrent sentences, another type of problem arose from the arbitrariness of the connection between a set of words and a state of affairs, and from the fact that a state of affairs can be described in various ways, both totally and partially. That is, a sentence like "An ass runs" can be reinterpreted to mean "God exists", or it can be taken to refer to just one aspect of the state of affairs that it signifies, namely, that an animal is running. These possibilities could wreak havoc with an otherwise valid consequence, or at least with a consequence whose validity was dependent upon the semantical rather than the syntactical properties of its constituents; and various clauses were added to ensure that propositions retained their original and total significance.[28] Caubraith was more lengthy in his discussion than most. He required that the propositions appearing in a consequence should maintain their virtual, adequate and total significance in order to exclude the following three cases: (1) "A man runs, therefore an ass runs" when read as "Socrates runs, therefore an animal runs"; (2) "Socrates runs, therefore every man runs" where the consequent can be taken in its partial significance as meaning "Socrates runs"; and (3) "Socrates runs, therefore a man runs", where "Socrates runs" is taken to signify that God exists.[29] One type of inference which was deliberately ruled out by these new restrictions was that from "'P' is true" to "P". As Almain indicated, from "This is true, "Man is an ass"," one cannot infer that man is an ass, for the set of words in question might mean "God exists". He did not take it for granted that "'P'" must be the name of "P".

Before I leave the question of how to define a valid consequence, it should be noted that one other definition was sometimes appealed to, namely that a consequence is valid if and only if the contradictory opposite

of the consequent is repugnant to the antecedent. Both Paul of Venice and Paul of Pergula had offered this as a definition, and they were followed by Blanchellus Faventinus, while others, like Celaya and Soto, mentioned it as a possibility.[30] More frequently, it was appealed to as a criterion for a valid consequence, together with the rule that the contradictory opposite of the consequent entails the contradictory opposite of the antecedent,[31] a rule which Niphus claimed to be the regulative principle of all consequences.[32] Two problems are associated with the criterion of repugnance, the definition of repugnance, and the status of certain consequences in relation to it. Most usually, propositions were said to be repugnant if they could not be true (or false) together,[33] but Hieronymus of St. Mark pointed out that this is inadequate, for it will make "No proposition is negative" repugnant to any other proposition. He preferred the two alternative interpretations, that two propositions are repugnant if it is impossible for things to be as signified by them together,[34] and that they are repugnant if they form an impossible conjunction.[35] It was often added that both impossible and false propositions were repugnant to any other, whereas necessary and true propositions were not.[36] It was this fact which was appealed to in order to solve the problem of the status of consequences like "Only a father exists, therefore not only a father exists" (because there must be a child) or "Man is an ass, therefore man is not an ass."[37] In each of these cases the contradictory opposite of the consequent is identical with the antecedent, and it seems odd to say that identical propositions can be repugnant. However, it was decided that an impossible proposition is repugnant even to itself, and that no conjunction containing such a proposition can ever be consistent.

Having established the general conditions of validity, our authors went on to distinguish between two kinds of consequence, formal and material.[38] This division corresponds to that between inferences valid on syntactical grounds and those valid on semantical grounds, for a formal consequence was said to be one whose validity depended on the form of the propositions alone, whereas a material consequence was one whose validity depended on the presence of certain terms, or on a certain kind of antecedent or consequent. One could find examples of material consequences having the same form, only one of which was valid. For instance, "Man runs, therefore God exists" and "Man runs, therefore an ass exists" have the same form, but only the first is materially valid, since it has a materially necessary consequent.[39] Some time was devoted to the discussion of what it was for two propositions to be identical in form, and it was agreed that they must have the same copula or connective, the same quantity, quality and relation of terms, and the same acceptance or supposition of terms.[40] That is, "'Man' is a word", "Man is a species" and "Man is an animal" were not taken to be similar in form. The relationship between the two types of inference was not so frequently discussed. Strode and Cajetan of Thiene had remarked that all formal consequences were also material, for, Cajetan explained, if a consequence is valid because of its mode of arguing, it follows that the terms are also linked,[41] but it should be noted that they

had a special theory about formality, and that they did not accept the paradoxes of strict implication as formal consequences. As will be seen below, Eckius and Niphus both realized that a materially valid consequence could be turned into a formally valid consequence by the addition of an extra premiss,[42] but other authors did not consider this possibility.

The special view of formality which Strode and Cajetan had held was not without influence on the period with which we are concerned. Strode had said that a formally valid consequence was one in which the consequent was understood in the antecedent, and Cajetan in his commentary on Strode discussed the matter at some length.[43] He divided formally valid consequences into two kinds, those valid *simpliciter,* where the contradictory opposite of the consequent could not be imagined together with the antecedent without a contradiction, no matter what the terms, and those valid *secundum quid,* where this was not so in all cases. Some called this latter sort 'material consequence', he said, and commented that it was not clear what attitude Strode would have adopted, for his requirement that the consequent be understood in the antecedent could be interpreted in three ways. It might mean "in essendo" (one cannot be a man without being an animal), it might mean "in consequendo", or it might mean both. It was not necessary, he thought, for the consequent to be included in or presupposed by the antecedent, though this was sufficient for formal validity. The Cologne commentators, Greve, and John of Glogavia all seem to have accepted the claim that the consequent should be understood in the antecedent without comment, while Major added that a relationship of pertinence between the terms was required, and the author of the *Libellus Sophistarum* said that the antecedent formally contains the consequent just when verifying the antecedent verifies the consequent.[44] Enzinas, however, explicitly rejected this view.[45] It isn't worth anything, he said, for one can easily understand that a man is running without understanding that an animal is running.

The views of Strode and Cajetan obviously have a very close connection with those of Paul of Venice and his follower Paul of Pergula. They made a two-fold division of valid consequences into formal and material and then again into formal *de forma* and formal *de materia,* the last two divisions being those normally called 'formal' and 'material'.[46] In their interpretation a formally valid consequence was one in which the contradictory opposite of the consequent could not be imagined together with the antecedent without contradiction, and a materially valid consequence was one in which such a conjunction could be imagined, although it was in fact impossible. The example given was "God does not exist and some man does exist", which can be imagined to hold by atheists. Not all formally valid consequences were also valid *de forma,* and "Only a father exists, therefore not only a father exists" was cited as an exception.[47] Both Blanchellus Faventinus and Javellus reproduced these arguments exactly, though Javellus did not think that the "only a father" example was even formally valid.[48] It was not a popular view, and Dolz rejected it with scorn. What, he asked, does "imaginable with" mean—"apprehended with"

"assented to together", or what? In many cases of obviously formal consequences, like syllogisms, one can certainly both apprehend and assent to the conjunction of the negated consequent and the antecedent, so how can this be used as a criterion?[49]

Another division of valid consequences which was sometimes discussed was that into consequences valid *simpliciter*, consequences valid *per accidens*, and consequences valid *ut nunc*, though the terminology varied in the last two cases, and one occasionally finds them both subsumed under the general heading of *ut nunc*.[50] Celaya, Soto and Pardo gave very similar accounts of the matter. A consequence is valid *simpliciter* when at no time is it possible for the antecedent to be true (or to signify etc.) without the consequent also being true (or so signifying etc.); it is valid *per accidens* when the antecedent cannot now or in the future be true without the consequent although this would once have been possible; and it is valid *ut nunc* when the antecedent cannot be true without the consequent, things being as they are now.[51] The most common example of a consequence valid *per accidens* was "Adam did not exist, therefore God does not exist" where the antecedent is said to be impossible *per accidens*. The exact status of the consequences said to be valid *ut nunc* was a matter of some discussion. Pardo explicitly rejected them, saying that "John sits, therefore William runs" is not a kind of consequence used by logicians.[52] Almain said that "Adam did not exist, therefore God does not exist" is a valid *ut nunc* consequence, but that "Socrates runs, therefore Plato runs" is not, even if both are now running. Celaya, John of Glogavia and Major, on the other hand, did not seem to be worried by this kind of inference.[53] John of Glogavia accepted "A man is sitting, therefore a stick is in the corner" as valid *ut nunc*, and John Major accepted "John is a priest, therefore John is an ass."[54] (One assumes that he was not a priest.) One may well wonder in what sense the last two examples were inferences, but the relationship between other consequences and *ut nunc* consequences in any interpretation was unfortunately not much discussed. Celaya said that every consequence which is simply valid is also valid *ut nunc*, though if a consequence is simply invalid it does not follow that it is invalid *ut nunc*;[55] and Eckius said that an *ut nunc* consequence like "Every man is a thinker, therefore you are" was not really valid unless one added the premiss "You are a man."[56] Niphus identified consequences *per accidens* with material consequences, and said that a consequence which is valid *per accidens simpliciter* like "A man runs, therefore an animal runs" becomes valid *per se* or formally if a necessary premiss, "Every man is an animal", is added. A consequence valid *per accidens ut nunc*, like "Socrates runs, therefore something white runs" becomes valid *per se* with the addition of a true premiss, "Socrates is white." In this way, all valid material consequences can be reduced to valid formal consequences.[57]

One aspect of *ut nunc* inferences, their link with present existence, was discussed only by the author of the treatise attributed to Peter of Spain, and by the Cologne commentators. The former reported that some people tied a consequence *ut nunc* to the existence of the subject of the

consequent, as in "Man is an animal, therefore this man is an animal", but he added that this was invalid because the antecedent is necessary, whereas the consequent is a possible proposition and will be false after the man is dead. Apart from consequences which were valid *simpliciter*, he would accept as valid only those like "John exists, therefore Aristotle did exist" which can never become invalid, though they may once have been invalid.[58] The Cologne commentators, however, said that "Man is an animal, therefore Socrates is an animal" can be accepted as valid because, despite the modern contention that "Socrates is an animal" means "Socrates exists as an animal", all that is really entailed is a link between subject and predicate. Socrates is a man and an animal whether he exists or not. Like Hundt after them, they said that a simply valid consequence involved a perpetual connection of terms, and a consequence valid *ut nunc* a contingent connection of terms.[59] It is curious that neither these authors, nor any of the others, sought a solution to their problems in a closer examination of the temporal reference of propositions. To the contemporary reader it may well seem that "This man [at t_n] is an animal" or "Plato is running [at t_n]" are just as eternally true or false as "Aristotle did exist". Perhaps part of the trouble lay in the identification of propositions with occurrent sentences, for it is only when a clear distinction is drawn between sentences and propositions, that one can argue that a group of sentences about the past, present and future can be used to express the same eternally true proposition.

How logicians distinguished between formal and material consequence often had a close bearing upon their classification of the so-called paradoxes of strict implication, that is, that from an impossible proposition anything follows, and that a necessary proposition follows from anything. Some people listed them without comment,[60] while others mentioned that they followed because of the criterion of repugnance, the impossible being repugnant to anything.[61] Clichtoveus offered one of the most succinct arguments when he said that if one takes an antecedent A and a consequent B, the only reason for denying the inference of B from A is that A is true and B is false. If A is true by hypothesis, then it can be true. But in the case of the first paradox, A is assumed to be impossible, and it is contrary to the definition of 'impossible' to say that it is possible for an impossible proposition to be true. Hence the impossible implies anything. By a parallel argument, if B does not follow from A this is because A is true and B false. Yet it is contradictory to say that it is possible for a necessary proposition to be false.[62] Other authors took the distinction between formal and material consequence, and showed that one could have two corresponding types of paradox.[63] From the formally impossible, which either is or implies a formal contradiction, anything follows by a formally valid consequence; but from the materially impossible, like "God does not exist", anything follows by a materially valid consequence. Niphus alone added that from a false proposition anything follows by a consequence valid *ut nunc*, for either an arbitrary proposition follows immediately, or if it does not, one can produce a proof by assuming the true proposition which is

the contradictory opposite of the original false proposition.[64] In the second case, that of the materially impossible, he noted similarly that one must assume the materially necessary proposition -P to cover the case where Q does not otherwise follow from P. For all three cases he offered the standard proof:[65]

$$\begin{array}{rl} & \ulcorner P \,.\, \text{-}P \urcorner \rightarrow P \\ & P \rightarrow \ulcorner P \vee Q \urcorner \\ & \ulcorner P \,.\, \text{-}P \urcorner \rightarrow \text{-}P \\ & \underline{\ulcorner P \vee Q \urcorner,\ \text{-}P \rightarrow Q} \\ \therefore & \ulcorner P \,.\, \text{-}P \urcorner \rightarrow Q \end{array}$$

This proof yields a proof for the case of a necessary consequent by appeal to the principle that the contradictory opposite of the consequent implies the contradictory opposite of the antecedent.

Those logicians who believed that the consequent had to be understood in the antecedent for a consequence to be formally valid, tended to list the paradoxes as being only materially valid.[66] The author of the *Libellus Sophistarum* gave a typical account. There are, he said, three kinds of materially valid consequence, that whose validity depends on the terms employed, that where the antecedent is both impossible and irrelevant to the consequent, as in "Man is an ass, therefore the stick is in the corner," and that where the consequent is both necessary and irrelevant to the antecedent, as in "You run, therefore God exists." It was, of course, acknowledged that there were some formally valid consequences whose antecedents were impossible, like "Man is an ass, therefore man is an animal", but these held by virtue of some other rule.[67] Blanchellus Faventinus and Javellus, who had both adopted the definition of a formal consequence in terms of the contradictory opposite of the consequent not being imaginable together with the antecedent, also labelled the paradoxes as materially valid, though Javellus had doubts even about this.[68] Maiolus definitely objected, for, he said, he could not see how the impossible was imaginable together with anything, and he certainly did not wish to call the paradoxes formally valid.[69]

In at least three cases the paradoxes were denied to be formally valid even though the standard formal proof for the first was carefully set out. Sermonete, in his remarks on Strode, said that something could follow formally from the consequent of a valid consequence without so following from the antecedent, since in the case of the paradox "*c* is understood in *b*, *b* in *a*, but *c* is not understood in *a*."[70] The Cologne commentators said that even though every step in the derivation of *Q* from $\ulcorner P \,.\, \text{-}P \urcorner$ is formal, one cannot accept the last step. This is because $\ulcorner P \,.\, \text{-}P \urcorner$ can be taken in two ways, absolutely, as a virtual contradiction, or for the sake of the argument. In this proof it is accepted for the sake of the argument, and since both P and -P have thus been conceded, one cannot use one part of the formal contradiction to deny the other part. That is, $\ulcorner P \vee Q \urcorner,\ \text{-}P \rightarrow Q$ has to be rejected![71] Soto agreed that $\ulcorner P \,.\, \text{-}P \urcorner \rightarrow Q$ could be proved only if the contradiction were taken absolutely; and he also appealed to common usage

to support his doubts about the paradoxes: who would say that if you are a stone, it follows both that you are and that you are not?

The corollary of the interest in semantical problems which is displayed throughout our authors' discussion of the nature of valid consequence was a failure to develop any syntactically adequate system. They made no attempt to draw up a list of axioms and rules from which theorems could be derived; nor did they employ any symbolism, apart from the occasional use of A, B and C to indicate either propositions or terms. Nevertheless, a large number of rules are to be found in their works; and in the second part of this paper I shall list these rules in as orderly a manner as I can.

Part Two. When one is formalizing a logical rule like *modus ponens*, one can present it in at least four different guises:

1. As a theorem: '$(((p \mathrel{\supset} q) \,.\, p) \mathrel{\supset} q)$.' This uses the object language; and p and q are proposition letters.
2. As a meta-theorem: $\ulcorner(((P \mathrel{\supset} Q) \,.\, P) \mathrel{\supset} Q)\urcorner$. This uses the meta-language; and P and Q are metalinguistic variables which range over propositions.
3. As a rule: $\ulcorner P \mathrel{\supset} Q\urcorner$, $P \rightarrow Q$. This also uses the metalanguage.
4. As a meta-rule: $P \rightarrow Q$, $P \vdash Q$. This uses a meta-metalanguage.

As is well-known, methods 3 and 4 are the most appropriate for the texts I am concerned with. Accordingly I shall use '$\rightarrow$' as a metalinguistic sign for "formally implies", '$\Rightarrow$' as a metalinguistic sign for "materially implies" [in the mediaeval sense]; and '$\vdash$' as a meta-metalinguistic sign to indicate that from one valid consequence, another can be formed. I shall also use '=' to indicate mutual implication; and $\ulcorner P \circ Q\urcorner$ to indicate that P and Q are compossible or that P *stat* with Q. Similarly $\ulcorner\text{-}(P \circ Q)\urcorner$ will indicate that P and Q are repugnant or that they are not compossible. My other symbols need no explanation.

1. *General Rules.*

1.11 $P \rightarrow Q$, $\mathbf{T}'P' \vdash \mathbf{T}'Q'$
1.12 $P \rightarrow Q$, $\mathbf{F}'P' \vdash \mathbf{T}'Q' \vee \mathbf{F}'Q'$
1.13 $P \rightarrow Q$, $\mathbf{F}'Q' \vdash \mathbf{F}'P'$

Ex vero non sequitur nisi verum. Ex falso autem bene sequitur verum.[73]

Si aliqua consequente sit bona et antecedente sit verum consequens est verum. Ex qua sequitur correlario altera et aliqua consequentia sit bona et consequens sit falsum antecedens est falsum.[74]

1.21 $P \rightarrow Q$, $\Diamond P \vdash \Diamond Q$

Si antecedens alicuius bone consequentie fuerit possibile consequens erit possibile.[75]

1.221 $P \rightarrow Q$, $\text{-}\Diamond\text{-}P \vdash \text{-}\Diamond\text{-}Q$

Si antecedens alicuius bone consequentie sit necessarium consequens similiter est necessarium.[76]

1.222 $-\Diamond-Q \vdash P \rightarrow Q$

Necessarium sequitur ad quodlibet. Id est omnis consequentia cuius consequens est necessarium est bona.[77]

1.231 $P \rightarrow Q,\ -\Diamond Q \vdash -\Diamond P$

Impossibile non sequitur nisi ex impossibili.[78]

1.232 $-\Diamond P \vdash P \rightarrow Q$

Ex impossibile sequitur quodlibet: Omnis consequentia cuius antecedens est impossibile est bona.[79]

1.24 $P \rightarrow Q,\ (\Diamond Q\ .\ \Diamond-Q) \vdash -\Diamond P \vee (\Diamond P\ .\ \Diamond-P)$

Si consequens est contingens antecedens est contingens vel impossibile.[80]

1.31 $P \rightarrow Q \vdash \ulcorner-(P \circ -Q)\urcorner$

In omni consequentia bona et formali oppositum contradictorium consequentis repugnat antecedenti eiusdem consequentie.[81]

1.32 $P \rightarrow Q,\ -Q \vdash -P$

Si aliqua consequentia sit bona ex opposito contradictorio consequenti sequitur oppositum contradictorium antecedentis.[82]

1.41 $P \rightarrow Q,\ \ulcorner(P \circ R)\urcorner \vdash \ulcorner(Q \circ R)\urcorner$

Si aliqua consequentia est bona et aliquod stat cum antecedente, illud idem non repugnat sed stat cum consequente.[83]

1.42 $P \rightarrow Q,\ \ulcorner-(Q \circ R)\urcorner \vdash \ulcorner-(P \circ R)\urcorner$

Quicquid repugnat consequenti bone consequentie etiam repugnat totali antecedenti.[84]

1.51 $P \rightarrow Q, R \rightarrow P \vdash R \rightarrow Q$

Quicquid antecedit ad antecedens alicuius bone consequentie antecedit ad eius consequens.[85]

1.52 $P \rightarrow Q,\ Q \rightarrow R \vdash P \rightarrow R$

Quicquid sequitur ad consequens bone consequentie sequitur ad eius antecedens.[86]

1.6 $P \vdash P$

A synonymo ad synonymum: ut sic: est consequentia formalis.[87]

1.71 $\ulcorner P\ .\ Q\urcorner \rightarrow R \vdash \ulcorner-R\ .\ P\urcorner \rightarrow -Q$
1.72 $\ulcorner P\ .\ Q\urcorner \rightarrow R \vdash \ulcorner-R\ .\ Q\urcorner \rightarrow -P$

Si aliqua est bona consequentia: et antecedens est una copulativa ex opposito consequentis cum una parte illius copulative infertur oppositum alterius partis.[88]

1.81 $P \rightarrow Q, P = R \vdash R \rightarrow Q$
1.82 $\ulcorner P \cdot Q \urcorner \rightarrow R, \ulcorner P \cdot Q \urcorner = \ulcorner Q \cdot P \urcorner \vdash \ulcorner Q \cdot P \urcorner \rightarrow R$

Quicquid sequitur ex una propositione sequitur ex alia sibi equivalente.

This principle is applied to the syllogism, since the transposed premisses are equivalent to the original premisses.[89]

1.91 $P \rightarrow Q, R \rightarrow S, \ulcorner -(Q \circ S) \urcorner \vdash \ulcorner -(P \circ R) \urcorner$
1.92 $P \rightarrow Q, R \rightarrow S, \ulcorner (P \circ R) \urcorner \vdash \ulcorner (Q \circ S) \urcorner$

Quandocunque ad aliqua antecedentia sequuntur aliqua consequentia si consequentia repugnant et antecedentia repugnant.

Si ad aliqua antecedentia sequatur aliqua consequentia ad copulativam compositam ex antecedentibus sequitur copulativa composita ex consequentibus illis.

This second rule is appealed to in order to prove the first. It is argued that if Q and S are repugnant, then the consequent is impossible, and that if P and R are consistent, then their conjunction is possible. This would violate the rule that the possible does not imply the impossible. Hence if Q and S are repugnant, P and R must also be repugnant, and thus we get the first rule.[90]

1.10 $\ulcorner P \cdot -\Diamond -Q \urcorner \rightarrow R \vdash P \rightarrow R$

Si ad aliquam propositionem cum aliqua necessaria vel cum aliquibus necessariis sequatur aliquod consequens ad eandem propositionem sine appositione illius necessarie vel illarum necessarium sequitur idem consequens.

P must be either impossible or contingent. If it is impossible then it implies anything, including *R*. If it is contingent, the consequent must be considered. R cannot be impossible, because a possible conjunction cannot imply an impossible proposition. If *R* is necessary, it follows from anything including *P*. If *R* is contingent, it cannot follow from a necessary proposition and hence must follow from *P* alone.[91]

2. *Rules for Conditional Propositions.*

2.1 $\ulcorner P \mapsto Q \urcorner, P \rightarrow Q$

Arguendo a tota conditionali cum positione antecedentis ad positionem consequentis consequentia est formalis.[92]

2.2 $\ulcorner P \mapsto Q, \urcorner -Q \rightarrow -P$

Arguendo a tota conditionali cum destructione consequentis ad destructione antecedentis consequentia est valida.[93]

2.3 $\ulcorner P \mapsto Q \urcorner = \ulcorner -Q \mapsto -P \urcorner$

Ab una conditionali ad conditionalem quae composita sit ex opposito consequentis prioris pro antecedente et opposito antecedentis pro consequente consequentia est mutua.[94]

2.41 $\ulcorner P \backsupset Q \urcorner \rightarrow \ulcorner \text{-}P \vee Q \urcorner$
2.42 $\ulcorner \text{-}\Diamond\text{-}(\text{-}P \vee Q) \urcorner \rightarrow \ulcorner P \backsupset Q \urcorner$

A tota conditionali affirmativa ad unam disjunctivam compositam ex contradictorio antecedentis et consequente est bona et formalis consequentia . . . sed non oportet consequentiam e contrario valere nisi quando partes illius disjunctive formaliter contradicunt vel participant legem contradictoriam.[95]

It was noted that the disjunction in 2.41 must be necessary, because of the rule that a necessary proposition cannot imply a contingent proposition.[96]

2.51 $\ulcorner (P \,.\, Q) \backsupset R, \urcorner \ulcorner \text{-}R \,.\, P \urcorner \rightarrow \text{-}Q$
2.52 $\ulcorner (P \,.\, Q) \backsupset R, \urcorner \ulcorner \text{-}R \,.\, Q \urcorner \rightarrow \text{-}P$

Arguendo a tota conditionali cuius antecedens est una copulativa cum destructione consequentis cum positione unius partis talis copulative ad destructionem alterius partis consequentia est bona.[97]

2.61 $\ulcorner P \backsupset (Q \vee R) \urcorner, \ulcorner P \,.\, \text{-}Q \urcorner \rightarrow R$
2.62 $\ulcorner P \backsupset (Q \vee R) \urcorner, \ulcorner P \,.\, \text{-}R \urcorner \rightarrow Q$

Arguendo a tota conditionali cuius consequens est una disjunctiva et positione antecedentis et destructione unius partis talis disjunctive ad positionealterius consequentia est bona.[98]

2.71 $\ulcorner (P \,.\, Q) \backsupset (R \,.\, S) \urcorner, \ulcorner \text{-}R \,.\, P \urcorner \rightarrow \text{-}Q$
2.72 $\ulcorner (P \,.\, Q) \backsupset (R \,.\, S) \urcorner, \ulcorner \text{-}R \,.\, Q \urcorner \rightarrow \text{-}P$
2.73 $\ulcorner (P \,.\, Q) \backsupset (R \,.\, S) \urcorner, \ulcorner \text{-}S \,.\, P \urcorner \rightarrow \text{-}Q$
2.74 $\ulcorner (P \,.\, Q) \backsupset (R \,.\, S) \urcorner, \ulcorner \text{-}S \,.\, Q \urcorner \rightarrow \text{-}P$

Arguendo a tota conditionali cuius antecedens est una copulativa et consequens etiam cum destructione unius partis copulative consequentis cum positione unius partis copulative antecedentis ad destructionem alterius consequentia est bona.[99]

2.81 $\ulcorner P \backsupset Q \urcorner, \ulcorner R \backsupset P \urcorner \rightarrow \ulcorner R \backsupset Q \urcorner$

Ad omnem conditionalem sequitur alia conditionalis eiusdem consequentis cuius antecedens antecedit ad antecedens prime.[100]

2.82 $\ulcorner P \backsupset Q \urcorner, \ulcorner R \backsupset P \urcorner, \ulcorner Q \backsupset S \urcorner \rightarrow \ulcorner R \backsupset S \urcorner$

Arguendo a conditionali affirmativa ad alteram conditionalem affirmativam que sic se habet quod antecedens conditionalis que est consequens infert antecedens conditionalis que est antecedens, et consequens conditionalis que est antecedens infert consequens conditionalis que est consequens est bona consequentia.[101]

2.9 $\ulcorner P \backsupset Q \urcorner \rightarrow \ulcorner \text{-}\Diamond\text{-}(P \supset Q) \urcorner$

Arguendo a tota conditionali ad unam cathegoricam de ly necessario cadente supra totam illam conditionalem consequentia est bona.[102]

2.10 $\ulcorner P \backsupset Q \urcorner \rightarrow \ulcorner P \supset Q \urcorner$

A conditionali illativa ad conditionalem promissivam ex eisdem terminis compositam valet argumentum et non e contra.[103]

The promissory conditional was defined in purely truth-functional terms.[104]

3. *Rules for Conjunctions.*

3.1 $\ulcorner P \,.\, Q\urcorner = \ulcorner Q \,.\, P\urcorner$

Sed nihilominus in hypotheticis copulativis et disjunctivis potest esse conversio simplex penes transpositionem extremorum totalium: ut bene sequitur: homo est animal et deus est igitur deus est et homo est animal.[105] He suggests that the reverse also follows.

3.21 $\ulcorner P \,.\, Q\urcorner \rightarrow P$
3.22 $\ulcorner P \,.\, Q\urcorner \rightarrow Q$

A tota copulativa ad quamlibet eius partem principalis consequentia est formalis.[106]

3.31 $-P \rightarrow \ulcorner -(P \,.\, Q)\urcorner$
3.32 $-Q \rightarrow \ulcorner -(P \,.\, Q)\urcorner$

A destructione partis copulative ad destructionem totius est formalis consequentia.[107]

3.4 $\ulcorner P \,.\, Q\urcorner \rightarrow \ulcorner P \vee Q\urcorner$

A tota copulativa ad disjunctiva ex eisdem partibus constitutam est formalis consequentia.[108]

This is so because a conjunction implies each of its parts and these in turn imply a disjunction.[109]

3.5 $\ulcorner P \,.\, Q\urcorner \rightarrow \ulcorner -(-P \,.\, -Q)\urcorner$

Arguendo a tota copulativa affirmativa ad copulativam negativam cuius prima pars principalis contradicit prime parti copulative affirmative et secunda secunde est formalis consequentia.[110]

3.6 $\ulcorner P \,.\, Q\urcorner \rightarrow \ulcorner -(P \supset -Q)\urcorner$

A copulativa ad conditionalem negativam constitutam ex prima parte et contradictorio secunde partis talis copulative est bona consequentia.[111]

3.711 $\ulcorner P \,.\, Q\urcorner, P \rightarrow R \vdash \ulcorner P \,.\, Q\urcorner \rightarrow R$
3.712 $\ulcorner P \,.\, Q\urcorner, Q \rightarrow R \vdash \ulcorner P \,.\, Q\urcorner \rightarrow R$
3.721 $\ulcorner P \,.\, Q\urcorner, P \Rightarrow R \vdash \ulcorner P \,.\, Q\urcorner \Rightarrow R$
3.722 $\ulcorner P \,.\, Q\urcorner, Q \Rightarrow R \vdash \ulcorner P \,.\, Q\urcorner \Rightarrow R$

Quicquid sequitur formaliter ex aliqua parte copulativa sequitur et ex tota: et quicquid materialiter et materialiter similiter.[112]

3.811 $P, P \rightarrow P, P \rightarrow Q \vdash P \rightarrow \ulcorner P \,.\, Q\urcorner$
3.812 $Q, Q \rightarrow Q, Q \rightarrow P \vdash Q \rightarrow \ulcorner P \,.\, Q\urcorner$
3.821 $P, P \rightarrow P, P \Rightarrow Q \vdash P \Rightarrow \ulcorner P \,.\, Q\urcorner$

3.822 $Q, Q \rightarrow Q, Q \Rightarrow P \vdash Q \Rightarrow \ulcorner P \cdot Q \urcorner$
3.831 $P, P \rightarrow Q \vdash P \rightarrow \ulcorner P \cdot Q \urcorner$
3.832 $Q, Q \rightarrow P \vdash Q \rightarrow \ulcorner P \cdot Q \urcorner$
3.841 $P, P \Rightarrow Q \vdash P \Rightarrow \ulcorner P \cdot Q \urcorner$
3.842 $Q, Q \Rightarrow P \vdash Q \Rightarrow \ulcorner P \cdot Q \urcorner$

Tertia regula arguendo a parte principali copulative affirmative ad totam copulativam non oportet consequentiam valere: quandoque tamen tenet: sed hoc solum ut in paucioribus materialiter, ut sic arguendo Sortes currit ergo Sortes currit et deus est: ubi prima pars copulative que est consequens primam partem eiusdem formaliter infert. Secundam vero materialiter et ex consequenti totam copulativam materialiter inferri debet. Aliquando tenet formaliter ut puta ubi una pars utramque infert formaliter sicut hic Sortes currit ergo Sortes currit et Sortes currit vel non currit.

Correlarie sequitur quando una pars copulative infert utramque partem copulative materialiter vel unam materialiter et aliam formaliter illa pars totam copulativam materialiter infert: et si una pars copulative utramque formaliter inferat eadem totam copulativam formaliter infert et non alias quare hec de priori particula.[113] The last four rules are licensed by the other sources (see footnote).

4. *Rules for Disjunctions.*

4.11 $P \rightarrow \ulcorner P \vee Q \urcorner$
4.12 $Q \rightarrow \ulcorner P \vee Q \urcorner$

Arguendo a parte principali disjunctive ad totam disjunctivam consequentia est formalis.[114]

4.21 $\ulcorner -(P \vee Q) \urcorner \rightarrow -P$
4.22 $\ulcorner -(P \vee Q) \urcorner \rightarrow -Q$

A destructione totius disjunctive ad destructionem partis eius est formalis consequentia.[115]

4.3 $\ulcorner P \vee Q \urcorner = \ulcorner Q \vee P \urcorner$

See 3.1.

4.41 $\ulcorner P \vee Q \urcorner, -P \rightarrow Q$
4.42 $\ulcorner P \vee Q \urcorner, -Q \rightarrow P$

A tota disjunctiva: cum destructione unius partis principalis ad alteram partem principalem consequentia est valida.[116]

4.51 $-P \rightarrow \ulcorner -((P \vee Q) \cdot -Q) \urcorner$
4.52 $-Q \rightarrow \ulcorner -((P \vee Q) \cdot -P) \urcorner$

A destructione unius partis disjunctive ad destructionem antecedentis: cuius contradictoria alterius partis pars una fuerit: est formalis consequentia.[117]

4.611 $\ulcorner P \vee Q \urcorner, -\Diamond P, \ulcorner(\Diamond Q \,.\, \Diamond\text{-}Q)\urcorner \rightarrow Q$
4.612 $\ulcorner P \vee Q \urcorner, -\Diamond Q, \ulcorner(\Diamond P \,.\, \Diamond\text{-}P)\urcorner \rightarrow P$
4.621 $\ulcorner P \vee Q \urcorner, -\Diamond P, \ulcorner(\Diamond Q \,.\, \Diamond\text{-}Q)\urcorner \Rightarrow Q$
4.622 $\ulcorner P \vee Q \urcorner, -\Diamond Q, \ulcorner(\Diamond P \,.\, \Diamond\text{-}P)\urcorner \Rightarrow P$

Si sit aliqua disjunctiva affirmativa: cuius una pars est impossibilis et altera contingens ab illa disjunctiva ad partem contingentem est bona consequentia. Et si pars impossibilis sit materialiter impossibilis consequentia materialis erit et si formaliter impossibilis consequentia formalis erit.[118]

4.711 $\ulcorner P \vee Q \urcorner, -\Diamond\text{-}P, \ulcorner(\Diamond Q \,.\, \Diamond\text{-}Q)\urcorner \rightarrow P$
4.712 $\ulcorner P \vee Q \urcorner, -\Diamond\text{-}Q, \ulcorner(\Diamond P \,.\, \Diamond\text{-}P)\urcorner \rightarrow Q$
4.721 $\ulcorner P \vee Q \urcorner, -\Diamond\text{-}P, \ulcorner(\Diamond Q \,.\, \Diamond\text{-}Q)\urcorner \Rightarrow P$
4.722 $\ulcorner P \vee Q \urcorner, -\Diamond\text{-}Q, \ulcorner(\Diamond P \,.\, \Diamond\text{-}P)\urcorner \Rightarrow Q$

Si sit una disjunctiva cuius una pars est contingens et altera necessaria ad partem necessariam est bona consequentia de formalis si talis pars sit formaliter necessaria et materialis si materialiter fuerit necessaria.[119]

4.811 $\ulcorner P \vee Q \urcorner, P \rightarrow P, Q \rightarrow P \vdash \ulcorner P \vee Q \urcorner \rightarrow P$
4.812 $\ulcorner P \vee Q \urcorner, Q \rightarrow Q, P \rightarrow Q \vdash \ulcorner P \vee Q \urcorner \rightarrow Q$
4.821 $\ulcorner P \vee Q \urcorner, P \rightarrow P, Q \Rightarrow P \vdash \ulcorner P \vee Q \urcorner \Rightarrow P$
4.822 $\ulcorner P \vee Q \urcorner, Q \rightarrow Q, P \Rightarrow Q \vdash \ulcorner P \vee Q \urcorner \Rightarrow Q$

Et si petas quando valet consequentia a tota disjunctiva ad alteram partem et quando non pro illo ponitur tale documentum. Quando una pars disiunctive ex utraque infertur in bona consequentia a disiunctiva ad illam partem est bona consequentia et non alias. Et si illa pars ex una parte disiunctive inferatur materialiter et ex altera formaliter consequentia materialis est. Et si ex utraque inferatur formaliter consequentia formalis erit. Formaliter sequitur (Sortes currit et Sortes non currit) vel Plato disputat ergo Plato disputat.[120]

The sources also license the following:

4.831 $\ulcorner P \vee Q \urcorner, Q \rightarrow P \vdash \ulcorner P \vee Q \urcorner \rightarrow P$
4.832 $\ulcorner P \vee Q \urcorner, P \rightarrow Q \vdash \ulcorner P \vee Q \urcorner \rightarrow Q$
4.841 $\ulcorner P \vee Q \urcorner, Q \Rightarrow P \vdash \ulcorner P \vee Q \urcorner \Rightarrow P$
4.842 $\ulcorner P \vee Q \urcorner, P \Rightarrow Q \vdash \ulcorner P \vee Q \urcorner \Rightarrow Q$

5. *De Morgan's Laws.*

5.1 $\ulcorner P \vee Q \urcorner = \ulcorner\text{-}(\text{-}P \,.\, \text{-}Q)\urcorner$

Arguendo a disjunctiva affirmativa ad copulativam negativam compositam ex partibus contradicentibus consequentia est mututa.[121]

5.2 $\ulcorner P \,.\, Q \urcorner = \ulcorner\text{-}(-P \vee \text{-}Q)\urcorner$

A tota copulativa affirmativa ad disjunctivam negativam compositam ex

partibus contradicentibus partibus illius copulative consequentia est formalis . . . et mutua.[122]

5.3 $\ulcorner\text{-}(P \vee Q) = \ulcorner\text{-}P \,.\, \text{-}Q\urcorner$

A disjunctiva negativa ad copulativam affirmativam compositam ex partibus illius disiunctive consequentia est mutua.[123] The context makes it clear that *contradicentibus* is understood after *partibus*.

5.4 $\ulcorner\text{-}(P \,.\, Q)\urcorner = \ulcorner\text{-}P \vee \text{-}Q\urcorner$

A copulativa negativa ad disjunctivam affirmativam compositam ex partibus contradicentibus partibus illius copulative consequentia est mutua.[124]

NOTES

The bibliography should be consulted for details of titles and place and date of publication.

1. Enzinas, *Tractatus Syllogismorum,* fo.i^{vo}, said "syllogismus est consequentia bona et formalis . . . omnis consequentia formalis que non tenet gratia alicuius regule logicalis tenebit syllogistice." *Cf.* Greve, fo. lxxi.

2. Andreas Kesler, *De Consequentia Tractatus Logicus* (Wittenberg, 1623). See my paper, "Andreas Kesler and the later theory of consequence," *Notre Dame Journal of Formal Logic*, vol. XIV (1973), pp. 205-214.

3. The first three views were those of Ferebrich (or Ferrybridge), Strode, and Paul of Pergula. Of the fourth he said: "... est communis quam solum reputa veram de mente magistri dicentis consequentiam quod consequentia formaliter sumpta est propositio illativa consequentis et antecedente. et est simile ac si diceret Consequentia est propositio in qua coniunguntur plures cathegorice per notam illationis."

4. Johannes de Glogavia, fo. lxxviii, said "consequentia est oratio habens antecedens et consequens cum nota illationis illative tenta." *Cf.* Gebwiler; Greve, fo. lii^{vo}; [Mainz]; Breytkopf; Eckius, fo. c; Trutvetter, and others. The list of inferential signs comes from Major.

5. The truth conditions for a conditional proposition were commonly said to be the same as the conditions for the validity of a consequence; and this is the reason why consequences were often discussed in the context of hypothetical propositions.

6. Le Fèvre, p. 69: "Et summatim omnis bona consequentia est conditionalis vera et e diverso."

7. [Cologne], fo. xcix-xcix^{vo}.

8. Eckius, fo. c^{vo}. *Cf.* Glogavia, fo. lxxviii^{vo}; Major; [Cologne], fo. xcix.

9. Gebwiler.

10. Marsilius of Inghen, p. 201; Hieronymus of St. Mark.

11. Celaya, *Magne Suppositiones*.

12. E.g. Gebwiler: "Bona est in qua impossibile est antecedens esse verum sine consequente." *Cf.* Le Fèvre, p. 6vo, p. 7vo, p. 65vo.

13. Pardo, fo. x.

14. Almain: "antecedens non possit esse verum [consequente] existente falso." *Cf.* Le Fèvre, p. 65vo: "consequentia bona est consequentia: cuius antecedens esse verum consequente existente falso est impossibile."

15. Niphus, p. 11vo. *Cf.* Pseudo-Scotus, p. 287.

16. Niphus, *loc. cit.* "Propterea oporteret addere bonam esse consequentiam, quando impossibile est antecedens esse verum, & consequens falsum. Si illa simul formentur."

17. See Dorp; Eckius, fo. xviii and fo. c^{vo}; Enzinas, *Primus Tractatus;* Glogavia, fo. lxxix-lxxix^{vo}; Pardo, fo. x; [Cologne], fo. xcix^{vo}.

18. Celaya, *op. cit.*

19. The distinction between 'possible' and 'possibly-true' goes back at least as far as Buridan and Albert of Saxony. Paul of Venice remarked, p. 90, ". . . multae propositiones sunt possibiles quae non possunt esse verae, ut omnis propositio est falsa, nulla propositio est vera, hoc est falsum."

20. Celaya, *op. cit.* ". . . distinguo antecedens scilicet quod non possit dari antecedens verum et consequens falsum: vel quod non possit dari antecedens verum in propositio vera: et consequens falsum: id est propositionem falsum. . . . Consequentia bona est illa in qua non potest dari antecedens verum et consequens falsum: id est consequentia in qua non potest dari ita esse sicut significatur per antecedens: vel potest significari absque hoc quod detur ita esse sicut significatur vel potest significari per consequens secundum tales significationes secundum quas sunt antecedens et consequens. . . ." *Cf.* Soto, fo. xxiii.

21. Almain: "Consequentia bona est consequentia sic se habens quod qualitercunque significatur per antecedens impossibile est esse quin ita sit sicut significatur per consequens et ibi capitur esse ut se extendit ad fuisse/ad fore/ et ad possibiliter esse." *Cf.* [Cologne], fo. xcix; Glogavia, fo. lxxix^{vo}.

22. See I. M. Bocheński, *A History of Formal Logic,* translated and edited by Ivo Thomas (Notre Dame, Indiana, 1961), p. 196 and p. 200. *Cf.* Cajetan, p. 34; Marsilius of Inghen, p. 201vo.

23. Caubraith, fo. lxviii. "Dicitur quod ad consequentiam requiritur et sufficit quod secundum aliquam significationem antecedentis formalis virtualis vel propositionis illi correspondentis impossibile sit ita esse sicut per ipsum vel ipsam significatur vel significari potest de significatione totali et adequata quin ita sit sicut per consequens formale virtuale vel propositionem correspondente significatur vel significari potest manente significatione totali et propositionali antecedentis et consequentis adequate eadem: et hoc dummodo consequens non habeat non significari sicut est annexum: nec tota consequentia se maleficet ubi ly impossibile non debet accipi prime intentionaliter et sincathegorematice . . . debet ergo determinare totum sequens ita quod reddat unam modalem compositam. . . ." *Cf.* Enzinas, *op. cit.*, fo. xx: "Et ad bonitatem consequentie sufficit et requiritur quod secundum aliquam significatione antecedentis vel propositionis ei correspondentis non possit ita esse sicut per

ipsum significatur vel significari potest manente eadem significatione totali quin ita sit sicut per consequens significatur vel significari potest: et hoc extra reflexivas. hoc est dummodo consequens non habeat non significare sicut est anneum nec tota consequentia se maleficet."

24. Caubraith, fo. lxviii^{vo}.

25. Pardo, fo. ix^{vo}; Enzinas, *op. cit.*, fo. xx; Soto, fo. xviii; Hieronymus of St. Mark.

26. Enzinas, *loc. cit.* Major appeals to the clause to exclude "No proposition is negative, therefore man is an ass"; "Every proposition is particular, therefore man is an ass" and "God exists, therefore some proposition is particular."

27. Enzinas, *loc. cit.*; Caubraith, fo. lxviii-lxviii^{vo}; Celaya, *op. cit.*

28. Almain; Enzinas, *loc. cit.*; Soto, fo. lxx^{vo}; Marsilius of Inghen, p. 201vo; Dorp; Pardo, fo. ix^{vo}; *Libellus Sophistarum Cantab.*

29. Caubraith, *loc. cit.*

30. Paul of Venice, p. 27vo; Paul of Pergula, p. 87; Celaya, *op. cit.*; Soto, fo. xxiii^{vo}.

31. E.g. Eckius, fo. ci and *Libellus Sophistarum.* *Cf.* Marsilius of Inghen, p. 203vo. It would be hard to find an author who did not mention these two principles, or speak of them as if they had some special status.

32. Niphus, p. 11. He attributes the view to Aristotle.

33. E.g. Greve, fo. liiii^{vo}. ". . . non possunt simul stare in veritate vel falsitate." *Cf.* Gebwiler. The *Libellus Sophistarum Oxon.* said that two propositions are repugnant if the verification of one falsifies the other.

34. *Cf.* Almain. "Propositiones non ideo dicitur repugnare quia non possunt esse vere simul sed quia qualitercunque per ambas significatur, impossibile est esse simul."

35. Eckius, fo. ci, and Major speak of impossible conjunctions. Pardo, fo. xxii^{vo} says that this definition is sufficient, but he prefers the one in terms of significance.

36. See [Mainz]; Trutvetter; Usingen; Gebwiler; Glogavia, Breytkopf.

37. Eckius, fo. ci; Niphus, p. 11; Major. The second example appears in Major and earlier in Paul of Venice, p. 88. Eckius has "Every plant is a stone, therefore no plant is a stone." "Only a father is discussed by Pseudo-Scotus, p. 289.

38. See Almain; Breytkopf; Celaya, *op. cit.*; Soto, fo. lxxiiii; Dolz, *Syllogismi*; Eckius, *loc. cit.*; Enzinas, *op. cit.*, fo. xx; Glogavia, fo. lxxviii; Gebwiler; Greve, fo. lii^{vo}; Hundt; Le Fèvre, p. 65vo; Niphus, p. 11vo; Pardo, fo. xi; and others. Almain said: "Consequentia materialis est. consequentia bona sic se habens quod est dabilis una similis forme que nihil valet ut Sortes currit ergo deus est. Consequentia formalis est consequentia bona sic se habens quod non potest dari una similis forma quin ita consequentia sit bona."

39. Celaya, *op. cit.*

40. Celaya, *op. cit.*; [Cologne], fo. ci^{vo}; Dolz, *op. cit.*; Eckius, fo. ci^{vo}; Le Fèvre, p. 65vo; Pardo, fo. xi.

41. Cajetan, p. 35. *Cf.* Strode, p. 2vo.

42. Eckius, fo. c^{vo}; Niphus, p. 11vo.

43. Cajetan, p. 35-35vo.

44. [Cologne], fo. ciiii; Greve, fo. lii^{vo}; Glogavia, fo. lxxix^{vo}; *Libellus Sophistarum Oxon.*

45. Enzinas, *Oppositionum Liber*, fo. l.

46. Paul of Venice, p. 27vo; Paul of Pergula, p. 87.

47. Paul of Venice, p. 28; Paul of Pergula, p. 89.

48. Javellus, pp. 225-227.

49. Dolz, *op. cit.*

50. Almain; Eckius, fo. c^{vo}; Glogavia, fo. lxxix^{vo}; Greve; Hieronymus of St. Mark; Hundt; *Libellus Sophistarum*; Major.

51. Celaya, *op. cit.*; Soto, fo. lxiiii-lxiiii^{vo}; Pardo, fo. x-x^{vo}. Soto wrote: "Dupliciter enim potest accipi consequentia ut nunc: Primo pro consequentia in qua non potest antecedens esse verum sine consequente rebus ut nunc se habentibus. . . . Alio modo potest accipi pro consequentia in qua non potest antecedens esse verum sine consequente, potuit tamen aut poterit, ut / si adam non fuit, deus non est."

52. Pardo, fo. x^{vo}.

53. Celaya, *op. cit.* "Consequentia bona ut nunc est consequentia in qua non potest dari antecedens verum et consequens falsum rebus se habentibus ut nunc: ut ista consequentia est bona ut nunc scilicet germanus est in aula ergo floretus est in aula."

54. Glogavia, *loc. cit.* "Consequentia ut nunc bona est in qua antecedens non potest esse verum sine consequente pro certo tempore ut homo sedet. ergo baculus stat in angulo. posito quod ita sit in re."

55. Celaya, *loc. cit.*

56. Eckius, fo. c^{vo}.

57. Niphus, p. 11vo. *Cf.* Pseudo-Scotus, pp. 287-288.

58. [Cologne], fo. c.

59. [Cologne], fo. c^{vo}.

60. E.g. Breytkopf; Enzinas, *Primus Tractatus*, fo. xx^{vo}.

61. Trutvetter; Gebwiler.

62. Le Fèvre, p. 71. "Si non sequitur a est ergo b est, sit gratia disciplina antecedens verum et consequens falsum. Cum antecedens a est sit verum per hypothesim: ipsum potest esse verum: et per positum est propositio impossibilis. igitur propositio impossibilis potest esse vera. quod est sue diffinitionis oppositum."

63. Almain; Eckius, fo. ci-ci^{vo}; Hieronymus of St. Mark; Pardo, fo. xxii-xxii^{vo}.

64. Niphus, p. 12. *Cf.* Pseudo-Scotus, p. 288.

65. *Cf.* Pardo, *loc. cit.* The proof is also found in Pseudo-Scotus, *loc. cit.*; Peter of Mantua; and Albert of Saxony, quoted Bocheński, *op. cit.*, pp. 204-205.

66. Cajetan, p. 35; Ferebrich, p. 93vo; Glogavia, fo. lxxxi; *Libellus Sophistarum.*

67. The example comes from Major. He compared it with "Man is an ass, therefore a stick is in the corner," where no term in the antecedent is pertinent to the consequent; and he remarked that this type of consequence is used at Oxford and Cambridge, but not at Paris.

68. Javellus, pp. 225vo-226.

69. L. Maiolus, *Epiphyllides in dialecticis* (Venice, 1497). He does not discuss consequences as such, but the paradoxes are the subject of chapter XIV which is headed "Increpat iuniores quod necessarium non sequatur ad quodlibet et nec ex impossibili sequatur quodlibet." In the same chapter he remarks "Omnis enim bona consequentia in syllogismum deduci potest qui consequens consequentiae directe concludat."

70. Strode, p. 3-3vo. "Ex ista via sequitur quod aliquid sequitur de forma ad consequens alicuius bone consequentie de forma: et non ad antecedens eiusdem: et c/ esse de intellectu b/ et b/ de intellectu a/ et tamen c/ non esse de intellectu a."

71. [Cologne], fo. ciiii.

72. Soto, fo. lxxiiii-lxxiiii^{vo}. He said of the claim that the impossible implies anything: "Hoc autem (fateor) non possum mihi suadere."

73. Almain. *Cf.* [Cologne], fo. cii^{vo}; Soto, fo. lxxiii^{vo}; Hieronymus of St. Mark; Major; Savonarola.

74. Almain. *Cf.* Celaya, *op. cit.*; Enzinas, *op. cit.*, fo. xx^{vo}; Major; *Libellus Sophistarum.*

75. Almain. *Cf.* Soto, *loc. cit.*; Glogavia, fo. lxxx^{vo}; Savonarola; Celaya, *op. cit.*; [Cologne], fo. cii^{vo}.

76. Almain. *Cf.* Soto, *loc. cit.*; Enzinas, *loc. cit.*; Celaya, *op. cit.*; Savonarola.

77. Celaya, *op. cit. Cf.* Hieronymus of St. Mark; Soto, *loc. cit.*; Enzinas, *loc. cit.*; Caubraith fo. cvii^{vo}. See earlier discussion.

78. Soto, *loc. cit.*

79. Celaya, *op. cit. Cf.* Enzinas, *loc. cit.*; Caubraith, *loc. cit.* See earlier discussion.

80. Almain. *Cf.* Soto, *loc. cit.*; Celaya, *loc. cit.*; Major.

81. Breytkopf. See earlier discussion.

82. Almain. See earlier discussion.

83. Hieronymus of St. Mark. *Cf.* Enzinas, *loc. cit.*; Eckius, fo. ci; Pardo, fo. xxiii^{vo}; Savonarola.

84. Hieronymus of St. Mark. *Cf.* Soto, *loc. cit.*; Eckius, *loc. cit.*; Major; Pardo, *loc. cit.*; Savonarola; Trutvetter.

85. Celaya, *op. cit.* Most authors give this rule.

86. Celaya, *op. cit.* Most authors give this rule.

87. Eckius, fo. ci^{vo}. *Cf.* Enzinas, *op. cit.*, fo. xx; Caubraith, fo. lxviii-fo. lxviii^{vo}; Celaya, *op. cit.*

88. Celaya, *op. cit. Cf.* Dorp; Glogavia, fo. lxxxix^{vo}; *Libellus Sophistarum*; Major; Savonarola; Trutvetter.

89. Enzinas, *Tractatus Syllogismorum*, fo. ii.

90. Hieronymus of St. Mark.

91. Hieronymus of St. Mark. Pardo, fo. xxiiii, gives the same proof.

92. Celaya, *Expositio.* Most authors give this rule.

93. Celaya, *op. cit.* Most authors give this rule.

94. Enzinas, *Primus Tractatus*, fo. xx. *Cf.* Soto, fo. lxxiiii; Le Fèvre, p. 71.

95. Caubraith, fo. lxx. *Cf.* Celaya, *op. cit.*; Dolz, *Disceptationes*; Soto, *loc. cit.*; Enzinas, *op. cit.*, fo. xx^{vo}; Major.

96. Enzinas, *Oppositionum Liber*, fo. l^{vo}.

97. Dolz, *op. cit. Cf.* Hieronymus of St. Mark; Le Fèvre, p. 72; Pardo, fo. lvi-lvi^{vo}.

98. Dolz, *op. cit.*

99. Dolz, *op. cit.*

100. Hieronymus of St. Mark. *Cf.* Pardo, fo. xlv; Le Fèvre, p. 71.

101. Caubraith, fo. lxx. *Cf.* Dolz, *op. cit.*; Enzinas, *Primus Tractatus*, fo. xx^{vo}.

102. Caubraith, *loc. cit. Cf.* Celaya, *op. cit.*; Dolz, *op. cit.*; Enzinas, *Oppositionum Liber*, fo. l^{vo}.

103. Caubraith, fo. lxii. *Cf.* Major: "ab omni conditionali in qua ly si tenetur illative ad unam in qua ly si tenetur promissive de eodem antecendente et consequente consequentia valet."

104. Caubraith wrote, fo. lxxi^{vo}: "ad veritatem huius si sortes veniet ad me dabo illi equum non requiritur quod sortes veniat ad me nec requiritur quod dem illi equum: etiam non requiritur quod non sit impossibile quod sortes veniat ad me quin ego dem illi equum: sed sufficit et requiritur quod si ponatur antecedens ponatur consequens: hoc est: quod si hec propositio est vera sortes venit ad me. hec est vera dabo illi equum." For other references and a discussion of the promissory conditional, see my paper "Strict and material implication in the early sixteenth century," *Notre Dame Journal of Formal Logic*, vol. XIII (1972), pp. 556-560.

105. Gebwiler.

106. Celaya, *op. cit.* Most authors give these rules.

107. Le Fèvre, p. 72. *Cf.* Dorp.

108. Le Fèvre, p. 73. *Cf.* Caubraith, fo. lxxxii; Dolz, *op. cit.*; Soto, fo. lxxxiii; Enzinas, *Primus Tractatus*, fo. xxi^{vo}.

109. Martinus de Magistris.

110. Caubraith, *loc. cit.*

111. Martinus de Magistris.

112. Dolz, *op. cit.* *Cf.* Caubraith, *loc. cit.*

113. Caubraith, *loc. cit.* *Cf.* Enzinas, *loc. cit.*; Gebwiler; and *Libellus Sophistarum* for accounts which are less elaborate and precise.

114. Celaya, *op. cit.* Most authors give these rules.

115. Le Fèvre, p. 72^{vo}.

116. Celaya, *op. cit.* Most authors give these rules.

117. Le Fèvre, p. 73.

118. Caubraith, fo. cviii. *Cf.* Le Fèvre, p. 74; *Libellus Sophistarum*; Gebwiler.

119. Caubraith, *loc. cit.* *Cf. Libellus Sophistarum*; Gebwiler.

120. Caubraith, fo. $cvii^{vo}$-cviii. *Cf.* Enzinas, *op. cit.*, fo. xxii; Gebwiler; *Libellus Sophistarum.*

121. Enzinas, *loc. cit.* *Cf.* Caubraith, fo. cviii; Soto, fo. lxxxiii; Enzinas, *Oppositionum Liber*, fo. lxi.

122. Celaya, *op. cit.* *Cf.* Dolz, *op. cit.*, Soto, *loc. cit.*

123. Enzinas, *op. cit.*, fo. lx^{vo}; *Cf.* Caubraith, fo. lxxxii; Hieronymus of St. Mark; Javellus, p. 241^{vo}; Le Fèvre, p. 74; Pardo, fo. $lxiii^{vo}$; Enzinas, *Primus Tractatus*, fo. xxi^{vo}.

124. Enzinas, *Oppositionum Liber*, fo. lx^{vo}. *Cf.* Dolz, *op. cit.*; Dorp; Hieronymus of St. Mark; Javellus, *loc. cit.*; Pardo, *loc. cit.* Le Fèvre, p. 73; and Enzinas, *Primus Tractatus*, fo. xxi^{vo}.

BIBLIOGRAPHY

Almain, J., *Consequentiae.* Paris (1508). No pagination.

Blanchellus Faventinus, M., *Commentum et quaestiones super logicam Pauli Veneti.* Tarvisii (1476). No pagination.

Breytkopf, G., *Parvorum logicalium opusculum.* Leipzig (1507). No pagination.

Cajetan of Thiene: see Strode.

Caubraith, R., *Quadrupertim in Oppositiones, Conversiones, Hypotheticas et Modales.* Paris (1510).

Celaya, Johannes de., *Magne Suppositiones.* Paris (1526). No pagination.

Celaya, Johannes de., *Expositio in primum tractatum summularum Petri Hispani.* Paris [1515?]. No pagination.

Clichtoveus, J., See Le Fèvre.

[Cologne]. *Textus omnium tractatum Petri Hispani . . . juxta processum magistrorum Colonie in bursa Montis regentium.* Cologne (1493).

Coronel, A. See Major.

Dolz, J., *Disceptationes super primum tractatum summularum cum nonnullis suorum terminorum intellectionibus.* Paris (1512). No pagination.

Dolz, J., *Syllogismi.* Paris (1512). No pagination.

Dorp, J., Commentary on J. Buridan, *Summula de Dialectica,* Lyon (1487). No pagination.

Eckius, J., *In Summulas Petri Hispani extemporia et succincta.* Augustae Vindelicorum (1516).

Enzinas, Ferdinandus de, *Tractatus Syllogismorum.* Paris (1526).

Enzinas, Ferdinandus de, *Primus Tractatus Summularum.* Salmanticae (1523).

Enzinas, Ferdinandus de, *Oppositionum Liber.* Paris (1528).

Ferebrich: See Strode.

Gebwiler, J., *Magistralis totius Parvuli artis logices compilatio.* Basileae (1511). No pagination.

Glogavia, Johannes de, *Exercitium super omnes tractatus parvorum logicalium Petri Hispani.* Argentine (1517).

Greve, Heinrich, *Parva Logicalia nuper disputata.* Leipzig (149-).

Hieronymus of St. Mark, *Compendium preclarum quod parva logica seu summule dicitur. . . .* Coloniae (1507). No pagination.

Hundt, Magnus, *Compendium totius logices.* Leipzig (1507). No pagination.

Javellus, Chrysostom, *Compendium Logicae.* Parisiis (1573).

Le Fèvre d'Étaples, J., *Jacobi Fabri Stapulensis artificiales nonnulle introductiones per Jodocum Clichtoveum in unum diligenter collecte.* Paris (1520).

Libellus Sophistarum. London (1501-1502). No pagination.

Libellus Sophistarum ad usum Cantabrigiensium. London (1510). No pagination.

Libellus Sophistarum ad usum Oxoniensium. London (c.1525). No pagination. The three books are identical in most sections.

[Mainz], *Modernorum summule logicales.* (1489?). No pagination.

Major, J., *Consequentie inchoate perfecte ab Anthonio Coronel.* Paris (c.1503). No pagination.

Marsilius of Inghem. *Compendiarius parvorum logicalium liber continens perutiles Petri Hispani tractatus priores sex et clarissimi philosophi Marsilii dialectices documenta.* Vienne Austrie (1512).

Martinus de Magistris, *Tractatus Consequentiarum.* Paris (1501). No pagination.

Niphus, A., *Super libros priorum Aristotelis.* Venetiis (1554).

Ockham, William, *Summa Totius Logicae.* Oxoniae (1675).

Pardo, H., *Medulla dialectices.* Paris (1505).

Paul of Pergula, *Logica and Tractatus de Sensu Composito et Diviso.* Edited by Sister Mary Anthony Brown. St. Bonaventure, New York, Louvain and Paderborn (1961).

Paul of Venice, *Logica.* Venetiis (1559).

Peter of Mantua, *Logica.* Padua (1477). No pagination.

Peter of Spain: for his treatise on consequences, see Cologne. The treatise has recently been translated by J. R. Mullally in *Tractatus Syncategorematum and Selected Anonymous Treatises.* Milwaukee, Wisconsin (1964).

Pseudo-Scotus, *Quaestiones super Librum I Priorum* in J. Duns Scotus, *Opera Omnia,* edited by L. Wadding, Vol. I. Lugduni (1639).

Savonarola, H., *Compendium Logice.* Florentiae (1497). No pagination.

Soto, Dominicus de, *Introductiones dialectice.* Burgos (1529).

Strode, R., *Consequentie Strodi cum commento Alexandri Sermonete. Declarationes Gaetani in easdem consequentias. Dubia magistri Pauli pergulensis. Obligationes eiusdem Strodi. Consequentie Ricardi de Ferabrich. Expositio Gaetani super easdem. Consequentie subtiles Hentisbari. Questiones in consequentias Strodi perutiles eximii artium doctoris domini Antonii Frachentiani Vicentini.* Venetiis (1517).

Trutvetter, J., *Breviarum dialecticum.* Erphordie (1500). No pagination.

Usingen, Bartholomaeus Arnoldi de, *Summa compendiaria totius logice.* Basileae (1507). No pagination.

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